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Asset Reshuffling, Capital Infusion and Optimal Capital Requirements

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ABSTRACT

This paper examines the problem of defining and computing optimal capital requirements for insured banks. Such requirements are optimal when they make an arbitrarily fixed deposit insurance premium actuarially fair for each bank. Optimal capital requirements for insured banks are determined via the duality between the fair insurance premium and capital-to-deposit ratio. When a bank's initial capital position is not optimal, a *capital infusion* or an *asset reshuffling* is required to restore capital to the optimal level. The determination of the optimal capital infusion, considering different *post-infusion* asset reshuffling strategies, is also discussed. Numerical examples are developed to illustrate the properties of the model.

Keywords: Deposit Insurance, Capital Infusion, Asset Reshuffling, Capital Requirements

Introduction

Without deposit insurance, risky banks could attract deposits only at higher interest rates, which limits the banks' excessive risk-taking. With deposit insurance, the deposits are treated as risk-free, and thus, no market-regulated costs restrain risk-

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taking behavior. As a result, fixed-premium deposit insurance creates moral hazard problems and destroys market discipline. The incentive to exploit the fixed-rate insurance system becomes serious as net worth diminishes.

To protect the deposit insurance fund, we must reduce the probability of bank failures and the size of risk exposure. A risk-based mechanism is the most popular proposal for deposit insurance reform. This mechanism can be implemented through either a risk-based premium or risk-based capital requirements. That is, either charging the insured banks a variable premium according to its asset risk or requiring extra capital to offset the insurer's additional risk exposure under a given fixed premium can achieve a risk-based mechanism. It is obvious that the optimal capital requirements for insured banks could be obtained by developing an appropriate model for risk-based premium.

Thus, this paper attempts to develop a fair premium model that offers two distinctive advantages. First, it considers the liquidity problems associated with demand deposits, which means that the deposit insurance contract may be triggered not only in bank insolvency, but also in bank being solvent but illiquid. Second, the model allows the exercise price of deposit insurance option to be different with the triggered price when the deposit insurer liquidates a bank. As will be shown in this paper, optimal capital requirements for insured banks can be derived easily via the duality between the fair insurance premium and capital-to-deposit ratio. When a bank's initial capital position is not optimal, a *capital infusion* or an *asset reshuffling* is required to restore capital to the optimal level. The determination of the optimal capital infusion, considering different *post-infusion* asset reshuffling strategies, is also discussed.

The remainder of this paper is organized as follows. Section 2 reviews the literature on the capital requirements. A derivation of deposit insurance pricing model is discussed in section 3. Section 4 uses the pricing model to determine the implicit optimal capital requirements and demonstrates its relation with capital infusion and asset reshuffling. In section 5, the estimation of parameters and numerical examples are developed to illustrate the properties of the model. The last section concludes this paper.

Literature Review

There is no doubt that an increase in capital can provide an increased *buffer* of reserve funds to absorb losses. However, the selection of asset portfolios may also be changed in response to capital requirements, and this creates indirect effects on bank's insolvency risk. Capital requirements are optimal if they cause any given premium to be actuarially fair. However, the impact of capital requirements on the riskiness of banks is still an ongoing debate.

1. Reconciliation with the Debate on Capital Requirements

Since capital requirements impose a tax on banks, some researchers argue that bank managers will adjust their asset portfolio to offset an increased capital requirement. Koehn and Santomero (1980) and Kim and Santomero (1988) argue that a stringent capital regulation via a fixed capital-to-asset ratio induces banks to increase their portfolio risks to circumvent the capital requirements. The portfolio's *asset reshuffling* depends on the risk aversion coefficient. Conservative institutions somewhat offset the capital restrictions. However, their more risky counterparts reshuffle the portfolios to a great extent. As a result, the total risk for the entire industry might increase in response to a stringent capital requirement. Thus, considering the reaction of the depository institutions to new regulatory stringency, the capital ratio constraint appears to be an inadequate tool to control the riskiness of banks and the probability of failures.

To readdress the debate on whether capital requirements may or may not be an adequate tool for controlling bank's asset risk, Gennotte and Pyle (1991) used the value maximization model to analyze the asset risk-leverage trade-off under the assumption of *decreasing* returns on risky assets. They have shown that deposit insurance leads banks to engage in inefficient investments if the loan cost function is increasing and convex in the level of investment and risk. An increased capital requirement may induce a decrease in the size of investment, but simultaneously increase the *per-unit* asset risk. In some cases, the probability of default and the expected deadweight liquidation costs increase with tighter capital requirements. Therefore, they suggest that an increase in bank capital is not a substitute for risk monitoring and control and may imply an increased need for such surveillance.

The papers reviewed above share three important features. One is the assumption that bank asset portfolio decisions are made under binding capital requirements. They focus on how a bank readjusts portfolios for an increase in its actual capital ratio. Second, whether the capital requirement is still met during the period between examinations does *not* affect the decision maker's payoff. That is, earlier studies assume that all banks comply with stricter capital regulation initially, but subsequent enforcement is not considered. Third, required capital does not optimally increase with asset risks.

Therefore, Kendall (1991) proposes a model that takes into account the risk-taking incentives *between* examinations and the *nonbinding* capital requirement situation. A capital requirement is not assumed to bind at either the beginning or end of the period. However, there are costs associated with noncompliance at the end of the period.[1] In this model Kendall has shown that *capital infusions* and decreases in asset risks may be wealth-increasing for equityholders since noncompliance costs alter their payoff functions. He concludes that an increase in capital requirement has an ambiguous effect on the incentive to increase both financial risks (the changes in leverage level) and operating risks (the changes in volatility of asset returns). Thus, stringent capital requirements may cause greater risk-taking at some points in time, but not imply a trend toward a riskier banking system.

2. Optimal Capital Requirements as an Implicit Premium

The value of deposit insurance is equal to the difference between the value of default-free deposit and the market value of uninsured deposit, which is a function of bank leverage and asset risk. Therefore, for any given insurance premium and relevant risk, the "adequate capital" is that at which the value of insurer's risk exposure equals predetermined level (Sharpe, 1978). That is, an optimal capital requirement should cause any given premium to be an actuarially fair value.

Similarly, by recognizing the impact of the FDIC's risk measurement errors on real sector equilibrium, Flannery (1991) asserts that if the insurance agency can observe bank risks without error, the risk-based premium is isomorphic to risk-based

[1] The cost of noncompliance could be a result of an *ex-post setting up* between the bank and the FDIC, as discussed by Kane (1986).

capital as a means of pricing deposit insurance., and thus both can make deposit insurance actuarially fair without distorting private-sector resources allocation. When the risk assessment involves substantial informational costs or asymmetries, deposit insurance pricing errors can be economically large and constitute a social cost. The socially optimal means of pricing deposit insurance indicates that both required capital and explicit premiums optimally vary with the measured asset risk. In other word, exclusive reliance on either risk-based capital or risk-based premium is not socially optimal. He suggests that the U.S. regulators should persist in the transition to risk-based capital requirements, and simultaneously introduce a complementary risk-based premium system.

As for valuing a fair deposit insurance premium, option pricing models provide a set of economic techniques. Merton (1977) was the first to apply option pricing model to evaluate the cost of deposit insurance and loan guarantees. Since Merton (1977), different option pricing model specifications were introduced.[2] By using equity as the modified call option on post-insurance assets, and relating the deposit insurance as a put option, Ronn and Verma(1986) argue that the problem of empirical estimation of risk and the deposit insurance premium is tractable. When time series data on the market value of equity and the book value of debt are available, the risk-adjusted premium can be calculated through an iteration procedure. Ronn and Verma assume that the insurance agency will infuse capital into the insolvent bank provided its assets value is still above certain limit. However, they do not incorporate the expected costs of the capital infusion into their pricing of insurance premium, which will obviously underestimate the fair deposit insurance premium.

3. Shortcomings of Previous Option Models

Although the use of option pricing model in analyzing deposit insurance premium has its merit, two critical drawbacks of previous models exist. First, since transaction services are an important function of depository financial intermediaries,

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[2] Merton's(1978) revised model explicitly takes into account surveillance cost and allows for random auditing times. Pennacchi(1987a) considered alternative regulatory policies and bank closure policies. Ronn and Verma(1986), Pennacchi(1987b), Crouhy and Galai(1991), and Duan et al.(1995) developed deposit insurance pricing model with stochastic interest rates. Duan and Yu(1999) employed GARCH option technique in determining the deposit insurance value.

a fair deposit insurance premium would never be derived without considering demand deposits. As demand deposits are random, it is inappropriate to model deposit insurance as an ordinary put option. Thus, a new type of model, other than ordinary put, should be considered.

To deal with the random deposit case, we should know the relation between liability and asset value. Note that exogenous changes in liability value induce identical changes in asset value. In other words, any random change of deposits will never render a bank insolvent provided the bank is solvent initially. However, banks are subject to liquidity constraints and there are some chances that big withdrawals may not be cleared. That is, though adverse clearing never causes insolvency, it may cause liquidity problems that lead to bank defaults. In this context, exercise of the deposit insurance contract could be triggered by not only the asset-side factor (*insolvency*) but also the liability-side factor (*illiquidity*). Previous studies have not considered bank failures due to liquidity problems. As a result, the premium calculated from an option-pricing model will be less than fair value of the insurance. Thus, this paper deals with the situation of liquidity problem by introducing a random deposit variable into an option-pricing model.

Second, previous studies failed to incorporate the liquidation cost of bank closure procedure into their models. The insurer pays off depositors at settlement if a liquidity problem occurs *or* if the bank is proved insolvent. In both cases, the bank is closed promptly and its assets are sold. Sale of the assets may incur a liquidation cost, which were not addressed in previous works. Ignoring the liquidation discount factor would certainly underestimate the deposit insurance premium.

This paper employs an alternative option-pricing model to deal with the second shortcoming. The Gap option model introduced in Boyle and Lee (1994) allows the trigger value of the option contract to be different from its exercise price. Unlike an ordinary option, the exercise price of which determines both the triggering condition for exercise of the option and also the amount of payoff, a gap option has a trigger value that is different from its exercise price.[3]

[3] Note that a gap option, with the same trigger value and exercise price, can be viewed as an ordinary Black-Scholes option.

The Fair Premium for a Deposit Insurance Contract

In the context of Chen and Osborne (2002), there are two independent factors that cause the changes in asset value, $\triangle A$. The first is the *autonomous factor*, $\triangle A_a$, which is the stochastic process of asset value that follows a Geometric Brownian motion. That is, $dA_a = \mu_A A dt + \sigma_A A dz_A$, where μ_A is instantaneous expected rate of return on assets; σ_A is instantaneous standard deviation per unit of time of the rate of return on assets and is assumed to be constant (since we study only short-term insurance contracts); and dz is a standard Wiener process. The other is the *induced factor*, $\triangle A_i$, which is the change in the liability-side. Since all the additions and withdrawals of deposits will be executed only at the time of clearing, the changes in deposit values will induce *equal* change in assets, e.g., $\triangle A_i = \triangle D$. The *liquidity problem* happens when the *net withdrawal* is greater than the bank reserve *plus* its credit line. The bank reserve is expressed as a proportion to initial asset value, αA_0. The credit line of a bank is assumed to be proportional to its initial net worth, $\beta (A_0 - D_0)$.

Let $\triangle D$ denote the change in deposits. That is, $\triangle D$ is negative when there is a net withdrawal. Since the change in deposits is lower bounded but not upper bounded, $-D_0 \leq \Delta D = (W-1)D_0 < \infty$, it is assumed that W follows a lognormal distribution. If the net withdrawal is greater than the bank's reserve, $\triangle D < -\alpha A_0$, then a borrowing is necessary. However, if the need of settlement funds is higher than its bank reserve plus credit line, $\triangle D < -[\alpha A_0 + \beta (A_0 - D_0)]$, then a liquidity problem arises and a prompt closure of the bank is assumed.

If a liquidity problem occurs *or* if the bank is proved insolvent, then the insurer pays off depositors at settlement. The bank is closed promptly and its assets are liquidated at discount. The payoff by the insurer is the difference between total liability and the asset's liquidation value at day 1. It can be expressed as $L - \rho A^*$, where L is the promised liability, $e^{rt}D_0$, A^* is the autonomous part of assets, $A_0 + \triangle A_a$, and ρ is the discount factor for asset liquidation. The conceptual structure of a one-period deposit insurance payoff schedule can be illustrated as Figure 1.

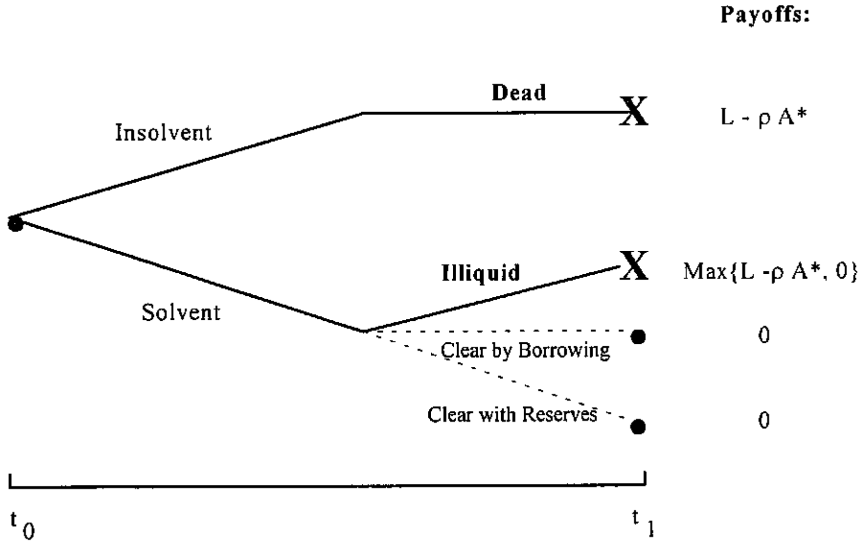

Figure 1 The deposit insurance contract is in-the-money at two possible cases. One is the case of insured bank being *insolvent*, and the other is the case of the insured bank being *solvent but illiquid*. Once the deposit insurance contract is exercised, the insurer pays off the difference between the bank's promised liability, L, and its liquidated asset value, ρA^*

There are two possible cases that may trigger exercise of the insurance contract. In the *insolvent* case, $A^*<L$, the insurer pays off the difference between the bank's promised liability and its liquidated asset value, L- ρA^*. In the second case, the payoff by the insurer is conditional on the bank being *solvent but illiquid*, i.e., $A^* \geq L \text{ and } \Delta D < -[\alpha A_0 + \beta(A_0 - D_0)]$ Since the bank is solvent, $A^*>L$, the liquidation value of its assets *may* or *may not* cover the promised liability, which means that it may have no loss to the insurer. That is, the payoff schedule of this case is Max{L- ρA^*, 0}.

By the *risk-neutrality* argument, Chen and Osborne (2002) have derived a fair premium of deposit insurance as follows:

$$P = e^{-r \cdot t} \cdot \begin{Bmatrix} prob\{A^* < L\} \cdot E\left[L - \rho A^* \middle| A^* < L\right] + \\ prob\{A^* \geq L \text{ and } \Delta D < -[\alpha A_0 + \beta(A_0 - D_0)]\} \\ \cdot E\left[Max\{L - \rho A^*, 0\} \middle| A^* \geq L \text{ and } \Delta D < -[\alpha A_0 + \beta(A_0 - D_0)]\right] \end{Bmatrix}. \tag{1}$$

After tedious manipulation[4], equation (1) reduces to the following explicit expression:

$$P=(1-\Lambda)\cdot\rho\cdot\mathbf{GP}\left(\mathrm{A_0},\frac{1}{\rho}\mathrm{L},\mathrm{L},\sigma,\mathrm{t_0},\mathrm{t_1}\right)+\Lambda\cdot\rho\cdot\mathbf{BP}\left(\mathrm{A_0},\frac{1}{\rho}\mathrm{L},\sigma,\mathrm{t_0},\mathrm{t_1}\right) \qquad (2)$$

where $\mathbf{GP}\left(\mathrm{A_0},\frac{1}{\rho}\mathrm{L},\mathrm{L},\sigma,\mathrm{t_0},\mathrm{t_1}\right)=\frac{1}{\rho}\mathrm{D_0}\cdot\mathrm{N}\left(-d+\sigma\sqrt{t}\right)-\mathrm{A_0}\cdot\mathrm{N}(-d),$

$$BP(A_0,\frac{1}{\rho}L,\sigma,t_0,t_1)=\frac{1}{\rho}D_0\cdot N\left(-h+\sigma\sqrt{t}\right)-A_0\cdot N(-h),$$

$$d=\frac{\ln\left(\frac{A_0}{D_0}\right)+\frac{\sigma^2\cdot t}{2}}{\sigma\sqrt{t}},\quad t=\mathrm{t_1}-t_0,\quad h=\frac{\left[\ln(\rho\cdot\frac{A_0}{D_0})+\frac{1}{2}\sigma^2 t\right]}{\sigma\sqrt{t}},t=t_1-t_0.$$

, and $\mathbf{GP}\left(\mathrm{A_0},\frac{1}{\rho}\mathrm{L},\mathrm{L},\sigma,\mathrm{t_0},\mathrm{t_1}\right)$ denotes the value of a Gap put option[5] with the exercise price being $\frac{1}{\rho}L$ and the trigger value being L, BP(.) is a Black-Scholes put option, $\Lambda(.)$ is the cumulative lognormal density functions of deposits and N(.) is the cumulative normal density function of assets.

That is, the premium consists of two parts. One is the expected loss from the bank being *insolvent*; the other is the cost incurred from the bank being *solvent but illiquid*, which is ignored by previous studies. That is, the value of the deposit insurance contract is a function of a Gap put option and an ordinary European put option. It consists of the expected losses from illiquidity as well as insolvency.

Chen and Osborne (2002) present insurance premiums estimated from the above

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[4] The derivations of all the equations are shown in Chen and Osborne(2002).

[5].The Gap option is introduced in Boyle and Lee (1994). Unlike an ordinary option, a gap option has a trigger value that is different from its exercise price. That is, the gap put option is exercised when the underlying asset value is less than the promised liability, $A^*<L$, and the payoff to the gap put option is $\frac{1}{\rho}L-A^*$. Note that a gap option, with the same trigger value and exercise price, can be viewed as a Black-Scholes option, i.e., $GP\left(A_0,\frac{1}{\rho}L,\frac{1}{\rho}L,\sigma,t_0,t_1\right)=BP\left(A_0,\frac{1}{\rho}L,\sigma,t_0,t_1\right)$.

model, and compares them with Merton's (1977) results. As we expect, the premiums estimated from this model are consistently larger than those from Merton's, since no liquidity problem is assumed in Merton's model. The result indicates that previous studies underestimate fair premium by overlooking liquidity problems. Other comparative-static analyses show that the fair insurance premium decreases in the bank reserves, the bank's credit-line limit in the interbank funds market, and the capital-to-deposit ratio, and increases in the asset volatility, the deposit volatility, and the liquidation discount factor.

The Implicit Optimal Capital Requirement Model

In a flat premium system the deposit insurance premium is fair only if the bank capital is adequate to set the riskiness of the bank to the predetermined level as suggested by Sharpe (1978). Given the duality between the insurance premium and capital requirement, it seems that we can invert the insurance premium model to determine optimal capital requirements for individual banks. However, we should be cautious in making this operation. Note that the inverse function exists *only if* a monotonic decreasing or increasing relation exists between the insurance premium and bank capital. Therefore, we should examine the existence of duality before applying the insurance premium model to derive the optimal capital requirements.

1. The Duality between Capital Requirements and Fair Insurance Premiums

Since the current deposit insurance system sets insurance premiums on per dollar of deposit basis, it is convenient for further discussions to express the premium as a ratio to deposits. Revising equation (3) into premium per dollar of deposit gives

$$p \equiv \frac{P}{D_0} = (1-\Lambda)\cdot\rho\left[\frac{1}{\rho}\cdot N\left(\sigma\sqrt{t}-d\right)-\left(\frac{A_0}{D_0}\right)\cdot N(-d)\right] + \Lambda\cdot\rho\cdot\left[\frac{1}{\rho}\cdot N\left(\sigma\sqrt{t}-h\right)-\left(\frac{A_0}{D_0}\right)\cdot N(-h)\right] \qquad (3)$$

To verify the duality relationship, a partial derivative of the insurance model with respect to capital-to-deposit ratio is examined. The result of the partial derivation, holding initial deposit unchanged, is shown in equation (4),

$$\frac{\partial p}{\partial k} = -\rho\cdot(1-\Lambda)\cdot\left\{(\frac{1}{\rho}-1)\cdot\frac{1}{\sigma\sqrt{t}}\cdot N'(-d)+\cdot N(-d)\right\}$$
$$-\rho\cdot\Lambda\cdot N(-h)-\Lambda'\cdot(\alpha+\beta)\cdot\left(\frac{BP}{D_0}-\frac{GP}{D_0}\right)<0, \quad (4)$$

where $k=\dfrac{Capital}{Deposit}=\dfrac{A_0-D_0}{D_0}$.

The above expression implies that the insurance premium is a monotonic decreasing function of the capital-to-deposit ratio. It confirms the existence of the duality between capital requirements and insurance premiums. The intuition of the result is obvious. An increase in capital can reduce the insurer's liability by providing extra buffers and reducing the probability of a bank failure. As a result, it reduces the fair premium of the deposit insurance. Given the relationship we now can determine optimal capital requirements for insured banks under a flat premium system.

2. The Implicit Optimal Capital Requirement Model

A capital requirement is optimal only when it makes a given fixed premium actuarially fair. Following Sharpe (1978) and Ronn and Verma (1989) we can equate equation (3) and the predetermined premium, $\overline{p}$, and then implicitly solve for the optimal capital ratio in the following equation (5)

$$(1-\Lambda)\cdot\rho\cdot\left[\frac{1}{\rho}\cdot N\left(\sigma\sqrt{t}-d\right)-(1+k)\cdot N(-d)\right]$$
$$+\Lambda\cdot\rho\cdot\left[\frac{1}{\rho}\cdot N\left(\sigma\sqrt{t}-h\right)-(1+k)\cdot N(-h)\right].=\overline{p}, \quad (5)$$

where $d=\dfrac{\left[\ln(1+k)+\frac{1}{2}\sigma^2 t\right]}{\sigma\sqrt{t}}, h=\dfrac{\left[\ln(\rho\cdot(1+k))+\frac{1}{2}\sigma^2 t\right]}{\sigma\sqrt{t}}$, and $\overline{p}$ is a predetermined premium per dollar of deposit and the other notations are the same. The solution of the above equation, k^*, is the required capital ratio to make the predetermined premium fair.

The solution of the above equation, k^*, is the required capital ratio to make the predetermined premium fair. Since $\overline{p}$ is the industry-wide premium, the result of the above optimal capital requirement for each individual bank is definitely different from the bank's initial capital level. Therefore, some capital adjustments would be needed before the deposit insurance contract is accepted.

If a bank's initial capital level is adequate, then it can either 1) request a premium reduction if possible under a variable premium system, or 2) adjust its capital position or asset portfolio to avoid subsiding other banks under a fixed premium system. However, if the insured bank's capital ratio is less than k^*, then the insurer would request an increase in capital to offset its additional risk exposure. Note that the *qualified capital* is defined as the difference between the asset value and deposit value. Adjustment of the capital ratio can be achieved by either retiring deposits or injecting fresh cash.[6] Since the insured deposits have no call provisions, an infusion of outside money is assumed if the capital ratio is below the required level.

Note that the implied deposit insurance premium is calculated according to the bank's initial asset risks, initial deposits and capital positions. After fresh cash infusion, the bank may reshuffle its asset portfolio. The adjustment action will affect the asset risks and, in turn, the implied premium and the optimal capital requirements. As a result, the amount of optimal capital infusion will vary with the riskiness of new assets. Consequently, computation of the capital requirement should be revised to incorporate the situations of possible asset adjustments in order to determine a required capital infusion when a bank's initial capital position is not optimal.

3. The Optimal Capital Infusion Model

To determine the optimal capital infusion considering the *post-infusion* asset reshuffling, we should discuss different investment strategies available to the banks. As suggested by Ronn and Verma (1989), there are three possible ways to reallocate new resources. The bank can 1) keep the asset risks unchanged by doing nothing on the existing portfolio, 2) reshuffle the asset portfolio by investing in new assets or

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[6] The new cash could be from new equity issued or any other *subordinated* debt issued.

adjusting portfolio weights, or 3) put all the proceeds of new capital into bank reserves. The amount of optimal capital infusion under different strategies is discussed as follows.

(1)Without Asset Reshuffling

The first simple case is no asset reshuffling. The bank invests the proceeds from new capital into the same asset portfolio and keeps the portfolio weights unchanged. In this way, the asset risks are the same as before. Therefore, the solution of equation (5) applies. The amount of the optimal capital infusion is the difference between the optimal capital ratio and the initial capital ratio times the initial deposits, i.e., $\left(k^* - k_0\right)\cdot D_0$.

(2)With Asset Reshuffling

Most likely, the bank would either invest the proceeds in new risky assets or adjust the existing portfolio weights. In both situations, the riskiness of the post-infusion portfolio would be changed. The standard deviation of the post-infusion portfolio depends on new portfolio weights, the standard deviation of infused portfolio and the covariance between the infused portfolio and existing assets. To simplify the discussion, we assume that an infused portfolio is purchased.[7]

The amount of optimal capital infusion is determined by equating the premium model and the predetermined premium, which is

$$\begin{aligned}(1-\Lambda)\cdot\rho\cdot\left[\frac{1}{\rho}\cdot N\left(\sigma_p\sqrt{t}-d\right)-\frac{\left(A_0+I\right)}{D_0}\cdot N(-d)\right]\\ +\Lambda\cdot\rho\cdot\left[\frac{1}{\rho}\cdot N\left(\sigma_p\sqrt{t}-h\right)-\frac{\left(A_0+I\right)}{D_0}\cdot N(-h)\right]=\overline{\text{p}},\end{aligned} \tag{6}$$

where $d=\dfrac{\left[\ln\left(\dfrac{A_0+I}{D_0}\right)+\dfrac{1}{2}\sigma_p^2 t\right]}{\sigma_p\sqrt{t}}$, $h=\dfrac{\left[\ln(\rho\cdot\left(\dfrac{A_0+I}{D_0}\right))+\dfrac{1}{2}\sigma_p^2 t\right]}{\sigma_p\sqrt{t}}$,

[7] The infused portfolio may consist of new assets and existing assets, which includes the case of adjusting portfolio weights.

$$\sigma_p^2 = \left[\frac{A_0}{A_0+I}\sigma_0^2 + \frac{I}{A_0+I}\sigma_I^2 + 2\left(\frac{A_0}{A_0+I}\right)\left(\frac{I}{A_0+I}\right)\sigma_{0I}\right],$$

I = the amount of capital infusion

σ_p, σ_0, σ_I = the standard deviation of the post-infusion portfolio, the initial portfolio, and the infused portfolio, respectively,

σ_{0I} = the covariance between the initial portfolio and the infused portfolio.

Previous studies[8] assert that banks would purchase assets that are riskier than the existing portfolio to circumvent the extra capital requirements imposed on them. In this regard, an increase in capital requirement may result in a higher probability of insolvency. Their assertion is true in a risk-insensitive premium system. With the risk-based mechanism incorporated in the implied premium calculation, such actions would require more capital infusions, which will deter the bank from taking excessive risks. Since the announcement of issuing new capital signals a bad news to the capital market (Myers and Majluf, 1984), it would be wise to keep the amount of capital infusion as low as possible for the pure capital requirement consideration.

(3)The Least Capital Infusion Case

As discussed above, an increase in the riskiness of the post-infusion portfolio would result in an extra capital infusion. It is because the buffer provided by the new capital is partly offset by the increased asset risks. Therefore, to have the least capital infusion the bank can keep all new capital as bank reserves.[9] In this case, the standard deviation of the post-infusion portfolio reduces to $\frac{A_0+I}{A_0}\cdot\sigma_0$. The required capital infusion is determined by implicitly solving equation (6) with the reduced asset volatility, which is less than that in previous cases.

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[8].See Koehn and Santomero (1980) and Kim and Santomero (1988).

[9] If the covariance between the infused portfolio and the initial portfolio is negative, then there *might* exist an investment strategy that requires a smaller capital infusion than that in all bank reserve case.

Numerical Examples

The main barrier to risk-adjusted deposit insurance system is the difficulty to quantify the riskiness of bank assets. Without the ability to calculate deposit insurance premium from known data, the implementation of such system will be infeasible. This section discusses the estimation of the parameters used in the present model. Later in this section a numerical example is provided to demonstrate how to determine optimal capital requirements and additional capital infusion if requested.

1. The Estimation of Parameters

As suggested by Osborne (1992), the simplest and most attractive way to circumvent moral hazard problems and make the constant volatility assumption plausible is to restrict the analysis to a very short-term deposit insurance contract. However, the short-term contract would be impractical if the fair premium could only be determined by examining the bank. The parameters used in the proposed deposit insurance equations can be categorized as observable/exogenous and unobservable parameters. Although some exogenous parameters, such as a bank's reserve, credit line, and changes in deposits, are observable from banks' daily operating reports, it is not public information to researchers. Nevertheless, these actual data are exclusively available to the banking governing agency or deposit insurer, which is applicable to the valuation of deposit insurance premium..

The following parameters are either observable or are assumed to be exogenous in the present analysis.

1) L: The promised liability is the initial value of deposit, D_0, plus incurred interest. Since the book value of initial deposits and the riskless rate are available, the promised liability is observable.

2) α, β: The ratio of bank reserves and the credit limit of the bank are assumed exogenous here. However, since the bank reserves are reported daily, it can be determined from actual data. It is assumed that the capital market knows the credit limit of each firm.

3) μ_ω, σ_ω: The location and scale parameters of the lognormal distribution of the changes in deposits are observable from the time series data. Since it is assumed

that there is no moral hazard problem embedded in the bank deposits, the distribution of the changes in deposits can be view as constant over time.[10] Thus, the parameters estimated from historical data should be unbiased.

4) ρ: The liquidity discount factor is assumed to be exogenous. Since the liquidation cost is not available until a bank is closed, a judgmental decision is necessary to the deposit insurance pricing. From numerical examples shown later, the deposit insurance premium is sensitive to liquidity discount factor.[11] Pennacchi (1987, a) also asserts that different closure policies would result in different equilibrium values of insurance. Therefore, to determine the optimal closure policy, and thus the discount factor, is a matter of practical concern.

Although both the value of assets, A_0, and its volatility, σ, are not directly observable, they are implicit in the value of the bank's securities (Ronn and Verma, 1986). As suggested by Black and Scholes (1973), the equity of a firm can be viewed as a call on the assets of the firm with striking price equal to maturity value of the debt. By this token, Ronn and Verma (1986) have shown that the calculation of the unobservable parameters in deposit insurance premium model is tractable if data on the market value of the bank's equity are available.

Asset volatility, the deposit-to-asset ratio, and the liquidation discount factor are three important parameters in the deposit insurance model. Chen and Osborne (2002) use numerical simulation and comparative static analyses with respect to those parameters, as well as other parameters, such as the deposit volatility, liquid asset ratios, and the line of credit limits, to demonstrates the effects of on the estimated insurance premium.

Their simulation results illustrate findings as follows. [12] First, the estimated fair

[10] Without deposit insurance depositors will concern about the riskiness of the bank assets, therefore, the changes in deposits may be partly affected by the asset risks. However, since deposits are fully insured by deposit insurance, the changes in deposits can be assumed to be independent of bank's asset risks.

[11] Chen and Osborne (2002) have shown negative partial derivative of premium with respect to liquidation factor, which indicates that if the insurer can employ the least cost resolution procedure to deal with a bank failure, then the fair premium would be lower.

[12] From the simulation results of Chen and Osborne (2002), it is shown that the value of asset volatility, the deposit-to-asset ratio, and the liquidation discount factor have more sensitive effects on implied deposit insurance premium than the effects of the rest of parameters.

premium increases with the deposit-to-asset ratio and asset volatility. Second, the premium decreases with the liquidation discount factor. Third, the insurance premium decreases with liquid assets and credit limits and increases with deposit volatility.

Since the deposit-to-asset ratio is exogenous and known to insurer, it is not a major concern here. Although the asset volatility is not directly observable, Ronn and Verma (1986) have shown that it is implicit in the value of the bank's securities. The critical parameter is the liquidity discount factor, which is assumed to be exogenous. However, since the liquidation cost is not available until a bank is closed, a judgmental decision is necessary to the deposit insurance pricing and thus this parameter is a matter of practical concern.

2. The Optimal Capital Requirement Example

Table 1 presents optimal capital requirements for insured banks under a flat insurance premium system. The standard deviations of asset return, σ, are selected as 0.006, 0.00225, and 0.046.[13] For simplicity it is assumed that the location parameter of the changes in deposits, μ_w, equals 0 and the scale parameter, σ_w, is set at 0.05. As mentioned above, the bank's reserve-to-initial-asset ratio, α, and the credit-line-to-initial-net-worth ratio, β, are assumed exogenous, which are set at 0.07 and 0.80 respectively. The liquidation discount factor, ρ, is selected as 0.80, 0.90, and 1.00.[14] The industry-wide deposit insurance premium is set at 1/12 of 1 percent level to determine a risk-based capital adequacy standard. Given arbitrarily selected parameters, an optimal capital requirement is obtained from a try-and-error iteration procedure of solving equation (5), setting the implied deposit insurance premium equal to the predetermined level of premium. As we expect, the optimal capital-to-deposit (debt-to-asset) ratio increases (decreases) with asset volatilities.

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[13].From the information provided by Marcus and Shaked (1984) and Ronn and Verma (1986) the ranges of σ were [0.009,0.045], [0.01, 0.037], [0.006, 0.033], and [0.007, 0.046]; the means were 0.023, 0.022, 0.018, and 0.016, respectively. We set the range of the standard deviation as [0.0060, 0.0460] and use 0.0225 as the base case following Kendall(1992).

[14].If the liquidation discount factor, ρ, equals 1, then the premiums estimated from both models are the same. It is because, without asset value discounted upon closure, the insurer suffers no losses when a bank is closed in the *solvent but illiquid* status.

From this example, we have shown the existence of duality between the implied premium and optimal capital requirement.

Table 1 The Optimal Capital Requirement

The optimal capital requirement, k*, measured by capital-to-deposit ratio, is obtained from an iteration procedure of solving equation (5), setting the implied deposit insurance premium equals to the predetermined level of premium, 1/12 of 1 percent on per dollar of deposit. The optimal debt-to-asset ratio, D/A*, is calculated by 1/(1+ k*). The standard deviations of asset return, σ, are selected as 0.006, 0.00225, and 0.046. The liquidation discount factor, ρ, is selected as 0.80, 0.90, and 1.00. The location parameter of the change in deposits, μ_w, is set at 0 and the scale parameter, σ_w, is set at 0.05. The bank's reserve-to-initial-asset ratio, α is set at 0.07 and the credit-line-to-initial-net-worth ratio, β, is set at 0.80.

	σ	k*	D/A*
Panel A: ρ=0.80	0.0060	0.0570895325	0.945993664
	0.0225	0.0673234850	0.936923073
	0.0460	0.1312336350	0.883990688
Panel B: ρ=0.90	0.0060	0.0404584955	0.961114743
	0.0225	0.0588573275	0.944414298
	0.0460	0.1199723650	0.892879173
Panel C: ρ=1.00	0.0060	0.0043168845	0.995701671
	0.0225	0.0320617025	0.968934316
	0.0460	0.0823322450	0.923907100

3. The Optimal Capital Infusion Example

We have shown the existence of duality between the implied premium and optimal capital requirement. Table 2 exhibits some examples to demonstrate this relation. The initial value of asset, A_0, and deposit, D_0, are set following Merton's(1977) numerical example. The standard deviations of asset return, σ, the location parameter of the changes in deposits, μ_w, and the scale parameter, σ_w, he bank's reserve-to-initial-asset ratio, α, and the credit-line-to-initial-net-worth ratio, β, are set at the same value in Table 1. The liquidation discount factors, ρ, is selected as 0.90. The initial capital-to-deposit ratio is defined as $k_0 = \dfrac{A_0 - D_0}{D_0}$.

Given an initial capital-to-deposit ratio, k_0, the optimal deposit insurance premium, p*, is calculated by solving equation (3). Given a fixed premium, 1/12 of 1 percent on per dollar of deposit, the optimal capital requirement, k*, measured by capital-to-deposit ratio, is obtained from a try-and-error iteration procedure of solving equation (5), setting the implied deposit insurance premium equal to the predetermined level of premium.

Table 2 Optimal Capital Infusions

The initial value of asset, A_0, and deposit, D_0, are set following Merton's(1977) numerical example. The initial capital-to-deposit ratio, k_0, is defined as $(A_0 - D_0)/D_0$. Given an initial capital-to-deposit ratio, k_0, the optimal deposit insurance premium, p*, is calculated by solving equation (3). Given a fixed premium, 1/12 of 1 percent on per dollar of deposit, the optimal capital requirement, k*, measured by capital-to-deposit ratio, is obtained from a try-and-error iteration procedure of solving equation (5), setting the implied deposit insurance premium equal to the predetermined level of premium. The standard deviations of asset return, σ, are selected as 0.006, 0.00225, and 0.046. The liquidation discount factors, ρ, is selected as 0.90 The location parameter of the changes in deposits, μ_w, equals 0 and the scale parameter, σ_w, is set at 0.05. The bank's reserve-to-initial-asset ratio, α, is set at 0.07, and the credit-line-to-initial-net-worth ratio, β, is set at 0.80.

	Initial Capital-to-Deposit Ratio k_0	Implicit Deposit Insurance Premium p*	Optimal Capital-to-Deposit Ratio k*	Infusion without Asset Reshuffling I_{no}	Least Capital Infusion I_{cash}
Panel A: σ = 0.0060					
A_0 = 100, D_0 = 90	0.1111111	0.0000003	0.04045849	0	0
A_0 = 100, D_0 = 95	0.0526315	0.0003644	0.04045849	0	0
A_0 = 100, D_0 = 100	0	0.0557738	0.04045849	4.045849535	4.045849535
Panel B: σ = 0.0225					
A_0 = 100, D_0 = 90	0.1111111	0.0000013	0.05885732	0	0
A_0 = 100, D_0 = 95	0.0526315	0.0016019	0.05885732	0.591446135	0.565049850
A_0 = 100, D_0 = 100	0	0.0615685	0.05885732	5.885732775	5.626087335
Panel C: σ = 0.0460					
A_0 = 100, D_0 = 90	0.1111111	0.0013346	0.11997237	0.797512865	0.714577750
A_0 = 100, D_0 = 95	0.0526315	0.0168364	0.11997237	6.397374695	5.731531550
A_0 = 100, D_0 = 100	0	0.0698326	0.11997237	11.997236550	10.747852835

Take a hypothetical bank in panel C with \$100 in assets and \$95 in deposits for example. The initial capital-to-deposit ratio is 0.0526315 and the implied premium under given conditions is 0.00168364. The optimal capital-to-deposit ratio is calculated from equation (5). If the initial capital-to-deposit ratio, k_0, is greater than optimal capital requirement, k^*, then no capital infusion is needed. However, if initial capital-to-deposit ratio, k_0, is less than optimal capital requirement, k^*,then additional capital must be infused. Given the value of 1/1200 as the fixed premium, the optimal capital-to-deposit ratio for the bank is 0.11997237, which is higher than its initial capital position. In this case a capital infusion is requested to meet the risk-based capital requirement or to bring the implied deposit insurance premium down to the predetermined level.

The dollar amount of optimal capital infusion without asset reshuffling is $6.39374695, which is the difference between the optimal capital-to-deposit ratio and the initial capital-to-deposit ratio times initial deposit, $\left(k^* - k_0\right) \cdot D_0$. However, the least dollar amount of capital infusion by keeping new capital in bank reserves is only $5.731531550. If the infused capital is invested in other risky assets, then the optimal capital infusion will be larger than that in least capital infusion case depending on the asset volatility of the post-infusion portfolio.

Conclusion

As discussed above, the risk-based deposit insurance system can be implemented either by introducing risk-based premium system or by requiring risk-based capital position. However, since capital regulation has a long history and the implementation is within regulators' discretionary power, the risk-based capital requirement system seems more appealing. A capital requirement is optimal when the implied premium obtained from the model is equal to the predetermined premium. If a bank's initial capital level is adequate, then it can either 1) request a premium reduction if possible under a variable premium system, or 2) adjust its capital position or asset portfolio to avoid subsiding other banks under a fixed premium system. However, if the insured bank's capital ratio is less than k^*, then the insurer would request an increase in capital to offset its additional risk exposure. Note that the *qualified capital* is defined as the difference between the asset value and deposit value. Either retiring deposits or injecting fresh cash can achieve adjustment of the capital ratio.[15] Since the bank has no call provisions on insured deposits, an infusion of outside money is assumed if the capital ratio is below the required level.

Note that the implied deposit insurance premium is calculated according to the bank's initial asset risks, initial deposits and capital positions. After fresh cash infusion, the bank may reshuffle its asset portfolio. The adjustment action will affect the asset risks and, in turn the implied premium and the optimal capital requirements. As a result, the amount of optimal capital infusion will vary with the riskiness of new assets. Consequently, as demonstrated in the paper, computation of the capital requirement should be revised to incorporate the situations of possible asset adjustments in order to determine a required capital infusion when a bank's initial capital position is not optimal.

The model discussed above permits a flexible deposit insurance system. Each bank with a different level of asset riskiness can determine its own capital position by paying the premium under this certain capital level. Alternatively, the insurer can request adequate capital to set the implied insurance premium equal to a predetermined explicit flat premium. Implementing this risk-sensitive mechanism would induce depository institutions to internalize the expected losses on their excessive risk-taking behavior. It is to be hoped that, with the moral hazard problem inherent in the current fixed premium system being resolved, the deterioration of the deposit insurance funds will come to an end.

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