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貨幣同盟內主權債務重整之探討

Indebted We Stand-Examining Debt Restructuring in a Currency Union

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摘要

本研究探討貨幣同盟之中主權債務重整之最佳方式，結果顯示直接減債優於延伸借款，因為後者比較有可能導致貨幣同盟的崩潰。本研究以 Gali & Monacelli (2008) 為基礎，採用貝氏觀念的馬可夫鏈蒙地卡羅方法，計算事後機率分配。分析顯示，歐元區的主權債務問題，如果貝氏觀念估算，其前景會比各界原來的預期更糟。但是，如果以減債紓困，會比延伸借款為佳，主要是因為後者會使消費、產出已及債務的市值變得更低。更重要的是，延伸借款的做法，迫使同盟內各國漸行漸遠。債務的回機率和收益率、財政負擔和通膨預期各項指標，都會使債務國離債權國越來越遠。即使同為債權國，在延伸借款的紓困措施下，越節省的國家預算壓力會越大，經濟規模越大的國家，通缩的風險會更高。

關鍵詞： 馬可夫鏈蒙地卡羅、貝氏模型、歐洲貨幣同盟、主權債務危機、債務重整

Abstract

This study examines the optimality of sovereign debt restructuring alternatives within a currency union. Its results suggest that upfront debt relief works better than extra lending, as the latter runs the risk of breaking up the union. Based on the model of Gali & Monacelli (2008), I adopt a Bayesian approach to calculate posteriors with the Markov Chain Monte Carlo (MCMC) method. I show in the analysis that, given the sovereign debt problem in Eurozone, the Bayesian approach projects an even more worrisome prospect than one would anticipate. Nevertheless, debt relief as a bail-out choice is superior to extra lending primarily because consumption, output and market value of debt are all lower under the latter option. More importantly, compared to debt relief, the lending alternative divides the union further apart. Debt recovery probability, as well as debt yields, fiscal burden, and inflationary expectations, drives debtor countries more away from the creditor ones. Even within the creditor group, extra lending would impose bigger budget hikes on more frugal states, in addition to subjecting larger economies to higher deflationary risks.

Keywords: Markov Chain Monte Carlo, Bayesian Model, European Monetary Union, Sovereign Debt Crisis, Debt Restructuring

1. Introduction

The new round of Greek sovereign debt crisis has renewed debates over debt relief versus further lending as an optimal resolution. Although accompanied by financial and economic reform requirements, the third Greece bail-out program maintains a lending extension to its current debt liability, which as an ultimate solution may arguably be suboptimal. Debt sustainability continues to be a central issue while IMF stresses adequate relief makes debt sustainable. It is imperative for authorities to be informed of the practically preferable long-term solution to the matter. Particularly, an empirical examination of the choice of extra lending over debt relief could bring about significant implications.

Bulow & Rogoff (1989) claim in their seminal article that “debts that are forgiven will be forgotten.” Before that, Krugman (1988) suggests that extra financing to a heavily indebted country distorts its policy, compared to forgiveness. Schwartz & Zurita (1992) show how forgiveness, as well as potential default penalty, is taken into account in the initial sovereign loan contract. Reinhart & Rogoff (2013) argue that debt overhangs need to be dealt with as in the cases of emerging markets, including debt forgiveness. As a related discussion on debt rescheduling, among others, Arellano & Ramanarayanan (2012) provide a model for the management of maturity. Das et al. (2012) document that pure rescheduling is far more common than outright face value write-offs. However, Cohen (1994) shows in his debt write-down simulation that a 50% write-off on a debt claim with an original face value of $1 would only lose merely 3.5%. While existing empirical works focus primarily on LDC (less developed countries) or HIPC (heavily indebted poor countries), this study attempts to examine empirically how effective debt forgiveness can be in mitigating the debt overhang problems in the Eurozone.

This study follows the model of Gali & Monacelli (2008), Roch & Uhlig (2014) and Dogra (2014) in which a currency union is made up of debtors who borrow to spend, and creditors who save and lend. I examine the optimality of sovereign debt restructuring alternatives by comparing economic and financial consequences of debt relief versus extra lending. Results of the study suggest that upfront debt relief works better than extra lending as the latter runs the risk of breaking up a the union. Parameters in this are time-varying and also involved in sequential relations with other variables. So I structure it in a state-space framework as Roch & Uhlig (2014), whose numerical analysis adopts a Bayesian approach with the Markov Chain Monte Carlo (MCMC) method which samples both the parameters and latent states. I show in the analysis that, given the sovereign debt problem in Eurozone, the Bayesian approach projects an even more worrisome prospect. Nevertheless, debt relief is a bail-out choice that is superior to extra lending. Consumption, output and market value of debt are all lower under the latter option. More importantly, compared to debt relief, the lending alternative divide the union further. Debt recovery probability, debt yields, fiscal burden and inflationary expectation separate debtor countries more apart from the creditor ones. Even within the creditor group, extra lending would impose the bigger budget hikes on the most frugal states, in addition to subjecting the larger economies to higher deflation risks.

The rest of the paper is structured as follows. Section II discusses the background of the existing model, related literature, and the theoretical model. Section III presents the empirical model employed, and results are given in Section IV. Section V gives a robustness analysis. Section VI concludes the paper.

2. A Financial Model of Currency Union

The setting of the model considered in this study is one similar to that in Eaton & Gersovitz (1981). Within a world with sovereign debt contracts, it is commonly known at date 1 that some debtor would default at date 2. Borrowers roll over old debt by issuing new debt, internalizing the effect of their borrowing decision on their bond price. Borrowers can reduce consumption and existing debt to raise the price of the remaining debt. Monetary authority can cut interest rate to raise bond demand from creditor countries to maintain full employment. However, they are limited by the There is a zero lower bound on interest rate restricting the authority. The currency union would be in a recession and output falls below production frontier when the bound is reached.

Under the constrained allocations in the economy, debt relief and debt rescheduling (extra lending) are then considered for the restoration to optimal allocations. Debt relief writes off a portion of a country's short-term debt while an extra lending policy is to increase a debtor country's long-term borrowing. With debt relief, a borrower could issue less debt without increasing consumption today. However, under an extra lending policy encourages borrowers to issue more debt and spend their transfer on date 1 consumption. If debtor countries spend most of the transfer on domestic goods and services, the creditor countries would longer be better off. A common argument against debt restructuring is that it gives countries an incentive to overborrow ex-ante, knowing that they will be bailed out. The model includes an ex-ante stage where countries decide how much to borrow and lend, taking into account that their debt may be restructured in the event of a recession.

2.1 Related Literature

The literature on debt issues in a currency union has become more available in the wake of European debt crisis. In Fornaro (2015) when the zero lower bound of interest rate is binding in a currency union, debt relief is Pareto improving. Debtor countries are forced to deleverage when they incur a shock to the borrowing constraint. As policy cannot bypass the borrowing constraint, extra lending policy or debt rescheduling is not possible. Official lending and debt rescheduling are considered as they are more common and politically feasible than principal write-offs. Forni & Pisani (2013) simulate a Dynamic Stochastic General Equilibrium (DSGE) model to address of sovereign debt restructuring in a monetary union, where sovereign spreads rise after restructuring, which is passed on to domestic households.

Regarding debt relief, one issue brought up by Pitchford & Wright (2012) is the incentive for a potential creditor, a holdout, to delay its agreement on restructuring by watching other creditors' move first. Krugman (1988) focuses on debt overhang by arguing that write-down may benefit creditors provided that increases the probability of the remaining debt being repaid. The argument brings along market-based debt reduction schemes with a debtor country buying back its own debt, which, according to Bulow & Rogoff (1988, 1991), benefits creditors only. Cole & Kehoe (2000) suggest debt relief may prevent self-fulfilling crises in multiple equilibria models. Roch & Uhlig (2014) argue that if an intervening agency guarantees some debt purchase at “good” equilibrium prices, then the move can reduce default events and thus make it cheaper for governments to borrow.

Aguiar & Amador (2013) show that sovereign debtors should write down short-term debt, but not their long- term debt. Writing down the latter is Pareto-improving, but cannot be implemented at equilibrium prices. The tradeoff between short and long-term debt is also discussed in Arellano & Ramanarayanan (2012), which stress that long-term debt hedges against fluctuations in interest rate spreads while short-term debt provides better incentives to repay.

2.2 A Theoretical Framework

I consider a model of currency union by Gali & Monacelli (2008), and the extension on debt restructuring by Roch & Uhlig (2014) and Dogra (2014). The original currency union is a closed system with member economies share identical preferences, technology, and market structure, but subject to imperfectly correlated shocks. There are creditor and debtor countries with governments maximizing household welfare from this period (t=1) till forever (t=∞). A debtor country issues a long-term bond, which is defaultable at this period, and short-term risk-free bonds maturing at t=2. Its budget constraints for the first two periods are

$$Q(d_2^i)(d_2^i - \bar{d}_2) + T_1^i = \bar{d}_1$$
$$T_2^i = d_2^i, \tag{1}$$

where $\bar{d}_1$ and $\bar{d}_2$ are the amounts of sovereign borrowing outstanding by a debtor country respectively at the start of period 1 and 2, d_2^i the amount of the long-term bond, issued by the debtor country. T_1^i and T_2^i are the transfers made to households in country *i*. $Q(d_2^i)$, the current value of the long-term bond, is given by

$$Q(d_2^i) = p(d_2^i)Q^{rf} \tag{2}$$

with *p*() being the pay-back probability in period 2, and Q^{rf} the price of short-term risk-free bond. The budget constraints of a creditor country are given by

$$\bar{d}_1 + T_1^i = Q_1(d_2^i - \bar{d}_2) + Q_1^{rf} a_2^i$$
$$T_2^i + p(d_2^i)d_2^i + a_2^i = 0, \tag{3}$$

where a_2^i is the face value of risk-free debt issued by the creditor country and due in period 2.

The common household utility is assumed to have the form of

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma} \; when \; \sigma > 1, else \; u(c) = \ln c \tag{4}$$

Given the requirements above, the initial debt level of $\bar{d}_1$, and the policy variables $d_2, \bar{d}_2, T_1^c$, T_1^D, and Q_1, the collection (c_1^C, c_1^D, a_2, Q^{rf}, y_1) is an equilibrium. A creditor country's problem is to choose its period 1 consumption, c_1^C, and period 2 fiscal surplus, a_2, in the following problem,

$$\max_{c_1^C, a_2} u(c_1^C) + \beta V(a_2)$$
$$s.t. \; c_1^C + Q^{rf} a_2 = y_1 + \bar{d}_1 - T_1^c, \tag{5}$$

where y_1 is period 1 net output and β is the one-period utility discount factor. *V* denotes the utility of the creditor country in a steady state where debtor countries default, and is expressed as

$$V\left(-d_2^i\right) = \frac{u(y^* - (1-\beta)d_2^i)}{1-\beta}, \tag{6}$$

and y^* is the steady state net output. Additionally, inflation would be such that

$$\frac{(1-Q^{rf})}{(1-y^*)}(\pi_t - 1) = a_2^i T_1^i \tag{7}$$

with π_t being the inflation at period *t*. A debtor country chooses c_1^D and d_2 to satisfy its budget constraint

$$c_1^D + \bar{d}_1 = y_1 + Q_1\left(d_2 - \bar{d}_2\right) - T_1^D. \tag{8}$$

The Union-wide government budget constraint is given by

$$Q_1 d_2 = Q^{rf} a_2 + T_1^c + T_1^D. \tag{9}$$

Moreover, the goods market clearing condition is

$$c_1^c + c_1^D = 2y_1. \tag{10}$$

The risk-free bond market clears according to

$$a_2 = p(d_2)d_2. \tag{11}$$

and

$$Q^{rf} \leq 1, \quad y_1 \leq y^* \tag{12}$$

applies with at least one strict equality.

The equilibrium can be employed to suggest whether debt relief or extra lending is more optimal for the monetary union. However, the complex policy variable structure makes it less tractable in a dynamic setting. Within the monetary union, given the observed choice variables, $(c_1^C, c_1^D, a_2, Q^{rf}, y_1)$, and the union-wide period 1 inflation, π_1, unobservable policy variables can be estimated in a state space framework. As the policies $(d_2, \bar{d}_2, T_1^c, T_1^D, Q_1)$ are involved in non-linear functions, their joint distributions are unlikely to be Gaussian. The parameters for the equilibrium are β, σ $\bar{d}_1$ and y^*, among others. Another parameter is, $\bar{d}_1^*$, the period 1 optimal risky borrowing which satisfies

(i) $Q^{rf} < 1, y_1 = y^*\ and\ Q^{rf}\ increases\ in\ \bar{d}_1\ if \bar{d}_1 < \bar{d}_1^*$;

(ii) $Q^{rf} = 1, y_1 = y^*\ \ if \bar{d}_1 = \bar{d}_1^*$;

(iii) $Q^{rf} = 1, y_1 < y^*\ and\ y_1\ increases\ in\ \bar{d}_1\ if \bar{d}_1 > \bar{d}_1^*$.

The other parameters is $p(d_2^i)$, the recover probability on debtor country's long term bond.

3. A Markov Chain Monte Carlo Model

Parameters in the model above are not only time-varying but also involved in sequential relations with other variables. So I structure it in a state-space framework as Roch & Uhlig (2014) which provides only numerical results instead of actual estimations using observed data. Although the sequential relationship among variables in the system above can be analyzed by Bayesian filtering algorithms such as Kalman Filter, the time-varying property of parameters has to be addressed. Therefore, I choose to estimate the model with an MCMC method, which samples both the parameters and the latent states. MCMC was applied in finance literature as early as in Jacquier et al. (1994), followed by Kim et al. (1998), to study stochastic volatilities, where MCMC is found to be superior to GMM and

QMLE. It was later reviewed in details in Johannes & Polson (2010) and Eraker et al. (2003) on the application of MCMC on continuous time finance models. In-depth elaborations are provided in Tsay (2010) and Hore et al. (2010). Xi et al. (2015) apply this method to estimate microstructure model of stock markets. It was also employed in the study of Value at Risk by Gerlach et al. (2011). Various applications and refinements in economics and finance areas are also reviewed in Creal (2012).

Classical filtering methods, such as Kalman filter, can generate inferences on state variable X, $p(X|\Theta,Y)$, given known parameters Θ and observed data Y. MCMC further provides the distribution of Θ through simulation. MCMC produces $p(\Theta,X|Y)$, and can be applied in nonlinear and non-Gaussian state models. MCMC is a unified estimation procedure that simultaneously estimates both parameters and state variables. In finance, this suggests MCMC can deal with inherent risks while estimating parameters or state variables. It simulates on conditional, rather than unconditional, distribution of state variables, which is computationally more efficient. The Monte Carlo simulation makes estimation procedure accurate after a large number of runs.

MCMC benefits greatly from the Hammersley-Clifford theorem, which says that a joint distribution can be characterized by its complete set of conditional distributions. In another word, $p(X|\Theta,Y)$ and $p(\Theta|X,Y)$ together characterize the joint distribution $p(\Theta,X|Y)$. From assumed distributions of the unobservable state variable X and parameter Θ, we can make initial draws, $X^{(0)}$ and $\Theta^{(0)}$, followed by drawing $X^{(1)} \sim p(X|\Theta^{(0)},Y)$ and $\Theta^{(1)} \sim p(\Theta|X^{(1)},Y)$. Continuing doing so generates a sequence of random variables $\left\{X^{(g)}, \Theta^{(g)}\right\}_{g=1}^{G}$, which is a Markov Chain converging to $p(\Theta,X|Y)$ as its equilibrium distribution. Dealing with two conditional densities, $p(X|\Theta,Y)$ and $p(\Theta|X,Y)$, is easier than with one higher dimensional joint density $p(\Theta,X|Y)$. This procedure can be conducted on direct draws with Gibbs Sampling (GS) according to Geman & Geman (1984) if $p(X|\Theta,Y)$ and $p(\Theta|X,Y)$ are in closed forms. Otherwise, a two-step procedure known as the Metropolis-Hastings (MH) algorithm, according to Metropolis et al. (1953) and Hastings (1970), is needed. Its first step samples a candidate draw from a proposal density to approximate the desired conditional distribution, and a decision is required to accept or reject the draw based on specified criteria. The MCMC algorithm is conducted by combining the GS and the MH procedures.

In the problem of this study, $X=\{\bar{d}_1, d_2, \bar{d}_2, T_1^c, T_1^D\}$, $\Theta=\{y^*, \bar{d}_1^*\}$ and $Y=\{c_1^C, c_1^D, a_2, Q^{rf}, y_1, \pi_1\}$. The discount factor β is assumed to be at 0.9, while the risk aversion σ takes on a moderately risk averse value of 2. The recovery probability $p(d_2^i)$ is assumed to follow a Gamma distribution to ensure a right-skewed distribution .The high dimensionality and nonlinearity of the system makes it extremely complex, hence unreliable, to make inferences on variables using traditional or maximum likelihood methods. Classical filtering procedures such as Kalman or particle filter are also likely to be restricted by the nonlinear structure. MCMC seems to be an ideal choice for estimation and inference making. The posterior conditionals in the GS step only requires draws from standard distributions such as normal, beta, gamma or binomial. I first choose an initial set of $\{y^*, \bar{d}_1^*\}^{(0)}$ and $\{\bar{d}_1, d_2, \bar{d}_2, T_1^c, T_1^D\}^{(0)}$, then sequentially draw

$$\{y^*, \bar{d}_1^*\}^{(1)} \sim p(\{y^*, \bar{d}_1^*\} | \{\bar{d}_1, d_2, \bar{d}_2, T_1^c, T_1^D\}^{(0)}, \{c_1^C, c_1^D, a_2, Q^{rf}, y_1, \pi_1\}) \quad (13)$$

and

$$\{\bar{d}_1, d_2, \bar{d}_2, T_1^c, T_1^D\}^{(1)} \sim p(\{\bar{d}_1, d_2, \bar{d}_2, T_1^c, T_1^D\} | \{y^*, \bar{d}_1^*\}^{(0)}, \{c_1^C, c_1^D, a_2, Q^{rf}, y_1, \pi_1\}). \quad (14)$$

The equilibrium distribution of the sequence

$$\{X^{(g)}, \Theta^{(g)}\}_{g=1}^{G}$$

is

$$p(\{y^*, \bar{d}_1^*\}, \{\bar{d}_1, d_2, \bar{d}_2, T_1^c, T_1^D\} | \{c_1^C, c_1^D, a_2, Q^{rf}, y_1, \pi_1\}).$$

A further refinement of the GS above is to carry out the draws in blocks of $\{y^*, \bar{d}_1^*\}$ and $\{\bar{d}_1, d_2, \bar{d}_2, T_1^c, T_1^D\}$ by applying the Hammersley-Clifford theorem. For instance, with respect to the parameter set $\{y^*, \bar{d}_1^*\}$, I can draw sequentially from

$$y^{*(1)} \sim p(y^* | \{ y^*, \bar{d}_1^*\}^{(0)}, \{\bar{d}_1, d_2, \bar{d}_2, T_1^c, T_1^D\}^{(0)}, \{c_1^C, c_1^D, a_2, Q^{rf}, y_1, \pi_1\}),$$
$$\bar{d}_1^{*(1)} \sim p(\bar{d}_1^* | \{ y^*, \bar{d}_1^*\}^{(0)}, \{\bar{d}_1, d_2, \bar{d}_2, T_1^c, T_1^D\}^{(0)}, \{c_1^C, c_1^D, a_2, Q^{rf}, y_1, \pi_1\}). \quad (15)$$

Similar sequential draws can be made on the state variable set $\{d_2, \bar{d}_2, T_1^c, T_1^D, Q_1, p\}$ as

$$\bar{d}_1^{(1)} \sim p(d_2 | \{d_2, \bar{d}_2, T_1^c, T_1^D\}^{(0)}, \{ y^*, \bar{d}_1^*\}^{(0)}, \{c_1^C, c_1^D, a_2, Q^{rf}, y_1, \pi_1\}),$$
$$d_2^{(1)} \sim p(d_2 | \{\bar{d}_1, \bar{d}_2, T_1^c, T_1^D\}^{(0)}, \{ y^*, \bar{d}_1^*\}^{(0)}, \{c_1^C, c_1^D, a_2, Q^{rf}, y_1, \pi_1\}),$$
$$\bar{d}_2^{(1)} \sim p(\bar{d}_2 | \{\bar{d}_1, d_2, T_1^c, T_1^D\}^{(0)}, \{ y^*, \bar{d}_1^*\}^{(0)}, \{c_1^C, c_1^D, a_2, Q^{rf}, y_1, \pi_1\}), \quad (16)$$
$$T_1^{c(1)} \sim p(T_1^c | \{\bar{d}_1, d_2, \bar{d}_2, T_1^D\}^{(0)}, \{ y^*, \bar{d}_1^*\}^{(0)}, \{c_1^C, c_1^D, a_2, Q^{rf}, y_1, \pi_1\}),$$
$$T_1^{D(1)} \sim p(T_1^D | \{\bar{d}_1, d_2, \bar{d}_2, T_1^c\}^{(0)}, \{ y^*, \bar{d}_1^*\}^{(0)}, \{c_1^C, c_1^D, a_2, Q^{rf}, y_1, \pi_1\}).$$

If a particular sequential parameter draw in (15) above turns out to be difficult to produce, then a second refinement procedure, the MH algorithm, can be invoked on

$$\Theta_i^{(g+1)} \sim p(\Theta_i | \Theta_i^{(g)}, \{\bar{d}_1, d_2, \bar{d}_2, T_1^c, T_1^D\}^{(g)}, \{c_1^C, c_1^D, a_2, Q^{rf}, y_1, \pi_1\}),$$
$$\Theta_i = y^* \text{or } \bar{d}_1^* \quad (17)$$

by specifying a recognizable proposal density $q(\Theta_i^{(g+1)} | \Theta_i^{(g)}, \Theta_{j\neq i}^{(g)}, X^{(g)}, Y)$ instead of $p()$ in the presentation above. The density is to be chosen such that its posterior ratio $\pi(\Theta_i^{(g+1)})/\pi(\Theta_i^{(g)})$ is apparently easy to find. The GS step described in (15) is then replaced by a candidate MH draw pending on acceptance according to prespecified probability. Specifically, the proposal draw $\Theta_i^{(g+1)}$ is accepted with the probability

$$min(1, \frac{\pi(\Theta_i^{(g+1)}) q(\Theta_i^{(g)} | \Theta_i^{(g+1)})}{\pi(\Theta_i^{(g)}) q(\Theta_i^{(g+1)} | \Theta_i^{(g)})}),$$

otherwise $\Theta_i^{(g+1)} = \Theta_i^{(g)}$. The proposal density $q(\Theta_i^{(g+1)} | \Theta_i^{(g)}, \Theta_{j\neq i}^{(g)}, X^{(g)}, Y)$ can be simplified further in two ways. One is to make the candidate draw $\Theta_i^{(g+1)}$ independent of $\Theta_i^{(g)}$ with

$$q\left(\Theta_i^{(g+1)}\middle|\Theta_i^{(g)},\Theta_{j\neq i}^{(g)},X^{(g)},Y\right)=q(\Theta_i^{(g+1)}|\Theta_{j\neq i}^{(g)},X^{(g)},Y) \tag{18}$$

The other simplification is to let $\Theta_i^{(g+1)}=\Theta_i^{(g)}+\varepsilon_t$, with ε_t being an independent mean zero error term. Additionally, in order to capture properties of the limiting distribution of the Markov Chain, $\{X^{(g)},\Theta^{(g)}\}_{g=1}^{G}$, a number of the initial draws would be discarded. To avoid possible correlation of nearby draws from the chain and improve convergence, as supported by Geyer (1992), only every *n*th draw will be made to 'thin' the chain.

Based on the principles mentioned above, an MH algorithm should follow GS, which is to be carried out first to make a draw of state variable *X*, given an initial value of Θ. However, for complex, nonlinear functions of *Y* governed by (5)~(12), the proposal density *q*() for parameters at each given step, though stochastically accepted or rejected according to MH, may turn out to be a poor one. A method of (FFBS) has been developed, and will hence be employed in this study, to draw states given the parameters (X |Θ), according to Fruhwirth-Schnatter (1994) and Carter & Kohn (1996). I use FFBS obtain $p(X^{(T)}\ |\{Y^{(1)},Y^{(2)},\dots,Y^{(T)}\})$ with Kalman filter. After that, the entire state $\{X^{(g)}\}_{g=1}^{T}$ can be drawn, given $p(X^{(T)}\ |\{Y^{(1)},Y^{(2)},\dots,Y^{(T)}\})$, using $p(X^{(t)}|\{X^{(g)}\}_{g=t+1}^{T};\{Y^{(g)}\}_{g=1}^{T})$. The latter reduces to $p(X^{(t)}\ |X^{(t+1)};\{Y^{(g)}\}_{g=1}^{T})$, as Fruhwirth-Schnatter (1994) and Carter & Kohn (1996) argue that the Markov structure makes them equivalent. If the underlying state-space is linear and normal or when the state is discrete, then the conditional distribution $p(X^{(t)}\ |X^{(t+1)};\{Y^{(g)}\}_{g=1}^{T})$ can be drawn utilizing information obtained in the forward Kalman filter steps. The state $\{X^{(g)}\}_{g=1}^{T}$ is discretized and then FFBS is applied. Detailed procedure is given in Appendix A.

To make draws of Θ given discretized draws of X, I will use the method of Griddy Gibbs (GG) described by Tanner (1996) and Tsay (2010). This method is introduced to cope with the conditional posterior distribution of nonlinear parameters, which could complicate steps in GS or MH. In essence, GG brings in grids to make random draws in GS provided the conditional posterior be univariate. This procedure is aim at discretizing the parameter such that its values are restricted to take on a grid of values. GG begins with a scalar parameter, such as y^* in this study, whose conditional posterior distribution is $f(y^*|X,\{\bar{d}_1^*\};Y)$ and is the proposed value of y^* given its current value. A grid of points are to be selected from a certain interval of y^*, say, $y_1^* \leq y_2^* \leq \cdots \leq y_m^*$. The conditional posterior density function of y^* is evaluated, which gives $d_j = f(y_j^*|X,\{\bar{d}_1^*\};Y)$ for j=1,…,m. These densities are then used to construct a cumulative distribution function $e_j = F(y_j^*|X,\{\bar{d}_1^*\};Y)$ and its inverse function $y_j^* = F^{-1}(e_j|X,\{\bar{d}_1^*\};Y)$. Draw from a discrete uniform (0,1) random variate and locate $\widehat{y_j^*}$ with a corresponding probability of $e_j, j = 1,\dots,m$. $\widehat{y_j^*}$ is to be the discrete GG random draw of originally proposed y^* in this study. Under the GG method, the normalization constant of the conditional posterior can be obtained directly from $\{d_j\}_{j=1}^m$. Then according to Baye's rule, the likelihood $f(Y|\Theta)$ can be derived exactly from $f(\Theta|X,Y)$ and the parameter prior $f(\Theta)$. The GG procedure seems to be randomizing the increment of y_j^*, similar to that in a random-walk MH. However, the finite grid defined under GG makes it different in appearance and also on the acceptance probability in the actual MH examination. What is left to be tuned is the interval $[y_1^*, y_m^*]$, which should be selected such that the probability of draws of y^* falling within an appropriate range. If that probability is too small, then the interval should be shortened, and vice versa. The GG procedure is to be applied on $\{y^*,\bar{d}_1^*\}$ one at time, following the completion on y^*. To avoid possible dependence arising from drawing single parameter as suggested by Geweke (2005), I then draw both parameters with GG. The approach will be the same as in the case of single parameter, except that the corresponding interval boundaries will be half of the previous values to make the joint proposal move of parameters more conservative.

4. Empirical Results

4.1 The Data

To estimate the latent state variables $\{\bar{d}_1, d_2, \bar{d}_2, T_1^c, T_1^D\}$ given the simulated parameters $\{y^*, \bar{d}_1^*\}$ and observed values of $\{c_1^c, c_1^D, a_2, Q^{rf}, y_1, \pi_1\}$, I obtain quarterly government finance data from the Statistical Data Warehouse of the European Central Bank. Data about national and general government accounts are extracted from the Eurostat database of the European Union. I select only the original twelve countries in the European Monetary Union when the common currency was physically issued. Quarterly data from the selected countries are available from the first quarter of 2001 till the first quarter of 2015. Summary statistics for the six observable variables, or their proxies, are given in Table 1.

Table 1 reports the quarterly observations, separately for the *Debtor* and the *Creditor* countries[1], from 2001Q1 to 2015Q1, covering the original twelve in the European Monetary Union. Among the six observable variables, $\{c_1^C, c_1^D, a_2, Q^{rf}, y_1, \pi_1\}$, c_1^D is the period 1 household consumption expenditure as a percentage, of the debtor countries, which, according to debt to GDP ratio, include Greece, Italy, Portugal, Belgium and Ireland. The creditor countries are the rest seven and c_1^C is the ratio of household consumption to GDP. a_2 is the period 2 face value of short-term risk-free debt issued by the creditor country, proxied by the increase of government liabilities and expressed as percentage of GDP. Q^{rf} reflects the EMU convergence criterion yield on a bond with roughly 10 years to maturity, but it is expressed as a value discounted from 1. y_1 is each country's quarterly national output indexed by the twelve-country average in 2000 and then transformed to fall withn an unit interval by a hyperbolic tangent function, while π_1 reflects one plus quarterly inflation. The division of creditor and debtor countries is according to the average government debt to GDP ratio within the data period, with the highest five assigned to the debtor country group.

[1] Although the data does not reveal explicitly the net borrowing or lending among the twelve countries, I assume those with heavier sovereign debt loads to be the debtor for simplicity.

Table 1 Summary Statistics of Observable Variables

Quarterly Averages 2001-2014	c_1^C	c_1^D	a_2	Q^{rf}	y_1	π_1
			Debtor Countries			
Greece		0.6651		0.9296	0.5521	1.0054
Italy		0.5921		0.9581	0.8021	1.0048
Portugal		0.6237		0.9495	0.5739	1.0050
Belgium		0.5003		0.9639	0.8859	1.0048
Ireland		0.4601		0.9549	0.9361	1.0046
Debtor Combined		0.5682		0.9512	0.7500	1.0049
			Creditor Countries			
Luxembourg	0.3225		0.0209	0.9688	0.9985	1.0051
Finland	0.4918		0.0438	0.9667	0.8925	1.0039
Netherlands	0.4502		0.0260	0.9669	0.9157	1.0047
Germany	0.5516		0.0266	0.9688	0.8892	1.0037
Spain	0.5649		0.0468	0.9587	0.7126	1.0056
Austria	0.5155		0.0320	0.9657	0.9019	1.0048
France	0.5280		0.0498	0.9658	0.8593	1.0038
Creditor Combined	0.4892		0.0376	0.9659	0.8814	1.0045

Note: Quarterly observations are obtained from Eurostat of the European Union and Statistical Data Warehouse of the European Central Bank from 2001Q1 to 2015Q1, covering the original twelve in the European Monetary Union. Among the six observable variables, $\{c_1^C, c_1^D, a_2, Q^{rf}, y_1, \pi_1\}$, c_1^D is the period 1 household consumption expenditure as a percentage of the debtor countries, which, according to debt to GDP ratio, include Greece, Italy, Portugal, Belgium and Ireland. The creditor countries are the rest seven and c_1^C is the ratio of household consumption to GDP. a_2 is the period 2 face value of short-term risk-free debt issued by the creditor country, proxied by the increase of government liabilities and expressed as a percentage of GDP. Q^{rf} reflects the EMU convergence criterion yield on a bond with roughly 10 years to maturity, but it is expressed as discounted values from 1. y_1 is quarterly national output indexed by the twelve-country average in 2000 and then transformed into an unit interval by a hyperbolic tangent function, while π_1 reflects one plus quarterly inflation. The division of creditor and debtor country group is according to the average government debt to GDP ratio within the data period, with the highest five assigned to the debtor countries.

Data source: this research

The comparisons between the debtor and the creditor countries suggest that in general, since EMU was established, debtor country's household consumption shares in GDP are on average 16% higher than those of the creditor countries. The latter also increases quarterly short-term liabilities for about 3.8% of GDP to finance government needs. The discount on a government bond, implied by yields complying with the Maastricht convergence criteria, is roughly 4.8% for a debtor country, and 3.4% for a creditor country respectively. The indexed output of a debtor is on average 17.5% lower than that of a creditor, after a hyperbolic tangent transformation, to restrict the respective figures within a unit interval, from an originally much wider range of numbers where debtor and creditor countries grew cumulatively in real terms from 2001 to 2014 by 9.73% and 66.88% respectively. The average implied annual inflation is more evenly distributed across EMU, with 1.96% for a debtor and 1.8% for a creditor.

Table 2 MCMC Estimation on the European Monetary Union

	Prior Mean	***Posterior Median***	***Standard Error***	***5% Quantile***	***95% Quantile***
			Parameter Estimates		
y^*	1.0000	0.9187	0.0477	0.8361	0.9915
$\bar{d}_1^*$	0.9000	0.8536	0.1521	0.5123	1.1995
			State Variable Estimates		
$\bar{d}_1$	0.9000	0.7462	0.1388	0.4060	1.0393
d_2	1.1000	1.2341	0.1792	0.6857	1.7699
$\bar{d}_2$	0.7000	0.5192	0.1163	0.2949	0.7337
T_1^C	0.2000	0.3142	0.0715	0.2140	0.4176
T_1^D	0.1000	-0.0731	0.0166	-0.0498	-0.0963

Note: MCMC procedures are carried out on the model (5)~(12). GG method is used in sampling the parameters $\{y^*, \bar{d}_1^*\}$ and the state variables $\{\bar{d}_1, d_2, \bar{d}_2, T_1^c, T_1^D\}$. FFBS procedure is employed to switch between parameters and states at each given step. MH procedure is also applied with the Gibbs algorithm. The FFBS procedure is iterated 25,000 times and the first 20,000 iterations are dropped as burn-in.

Data source: this research

Results from MCMC procedures carried out on my data are given in Table 2. Based on the model of (5)~(12), GG method is used in sampling the parameters $\{y^*, \bar{d}_1^*\}$ and the state variables $\{\bar{d}_1, d_2, \bar{d}_2, T_1^C, T_1^D\}$. FFBS procedure is employed to switch between parameters and states at each given step. MH procedure is also applied with the Gibbs algorithm. The FFBS procedure is iterated 25,000 times and the first 20,000 iterations are dropped as burn-in. Prior distributions of the parameters and the states are assumed to be independent of one another. To fit the expected empirical properties of parameters and state variables used in my model, I follow the convention of works in the literature of DSGE, specifically related to the discussions in Smets & Wouters (2003) and Lubik & Schorfheide (2007), among others. Beta distribution is employed for parameters or state variables bounded within a unit interval, such as y^*, T_1^C and T_1^D. For those who are positive but bounded by a small positive number, such as $\bar{d}_1^*$, $\bar{d}_1$, d_2, and $\bar{d}_2$, Gamma distribution is chosen for an approximation of negative skewness. Parameters of the prior distributions are calibrated such that their sample means and standard errors, after minor transformation on the original variables, match observations on related empirical variables.

The prior means of the two parameters are 1 and 0.9 respectively, reflecting the possible production frontier and roughly the 2001-2014 average EMU debt to GDP ratio of 0.98. The sampling results on parameters from the GG-within-FFBS method produced on average a posterior means of 0.9187 and 0.8356 for y^* and $\bar{d}_1^*$. The prior mean $\bar{d}_1$, the amount owed by the debtor to the creditor in the beginning, is set at 0.51 to match the difference between the debtor and the creditor groups on long-run average debt to GDP ratios. The posterior mean obtained from the Gibbs sampling is 0.7462, reflecting the fact that the debtor-creditor difference of debt to GDP ratio rose sharply from below 0.35 before 2005 to above 0.6 after 2010.

The estimates for the latent state variables, filtered by the FFBS algorithm, are reported in the bottom panel of Table 2, given the backward GG parameter sampling results in the top panel. The prior mean of d_2 is set at 1.1 to reflect the debtor group's average debt to GDP ratio, which is 1.29 during the last five years in the data set. The GG sampling yields a posterior mean of 1.3135 and a 95%

quantile of 1.5006, indicating the long-term debt born by the debtor group could possibly compare with the group's current debt to GDP ratio in 2014. To acknowledge the possibility of deleveraging by governments within the union, $\bar{d}_2$ is assumed to take on a prior mean of 0.7. The posterior mean turns out to be lower than the prior, projecting, under (2), possibly a higher than expected rollover on long-term borrowing to be conducted by the debtor group driven by anticipated low financing cost $Q(d_2)$, or a lower than expected government transfer T_1^D under austerity, whose posterior amounts to 7.3%. From (2), the implied yield on $Q(d_2)$ is approximately 6% given the sampled means of $\bar{d}_1$, d_2 and $\bar{d}_2$ in Table 2. Taxes for the creditor group, T_1^C, is assumed to have a prior of 0.2, given that average government consumption of the group averaged 20.3% of GDP within the data period. The posterior appears to be at 0.3142, reflecting pressure from a long-term borrowing rollover by $\left(d_2^i - \bar{d}_2\right)$. The prior for the debtor group's government transfer to households, T_1^D, is assumed to reflect a lower tax burden at the value of 0.1, given the group's average government consumption at 19.2% of GDP. But the sampled posterior distribution exhibits a mean of -0.0731, indicating a transfer to households, compatible also with a projected high value of long-term borrowing rollover according to (8).

Table 3 Simulations of EMU Economic Prospects Given Debt Restructuring Alternatives

	c_1^c		c_1^D		$Q(d_2)$		y_1		π_1	
	Mean	*Std. err.*	*Mean*	*Std. err.*	*Mean*	*Std. err.*	*Mean*	*Std. err.*	*Mean*	*Std. err.*
							Without Debt Restructuring			
Creditor	0.4749	0.0578					0.8102	0.1085	1.0041	0.0268
Debtor			0.5759	0.0694	0.8858	0.2526	0.6726	0.0877	1.0046	0.0273
							Under 20% Debt Relief			
Creditor	0.4923	0.0623					0.8691	0.1148	1.0044	0.0262
Debtor			0.5791	0.0707	0.8994	0.2559	0.7439	0.0951	1.0048	0.0273
							Under 20% Extra Lending			
Creditor	0.4491	0.0575					0.7995	0.1099	1.0031	0.0251
Debtor			0.5866	0.0762	0.8709	0.2492	0.6550	0.0869	1.0054	0.0263

Note: This table reports statistics from simulations of 10,000 runs on policy variables $\{c_1^C, c_1^D, Q(d_2), y_1, \pi_1\}$ whose MCMC parameters and state estimates are given in Table 2[a]. The scenario without debt restructuring is where no perturbation is imposed on the original system. The second scenario under debt relief is one where $\bar{d}_1$ in (8) is reduced exogenously, so that c_1^D there can be increased. Empirically, this is carried out by lowering the mean of $\bar{d}_1$ by 20%, from 0.7462 to 0.5967, in its simulation. The last scenario under extra lending is one where d_2, the debtor country's long-term borrowing, in (8) is raised exogenously, which allows c_1^D to increase potentially. Empirically, the mean of d_2 is dropped also 20%, from 1.3135 to 1.0508, in the following simulations.

[a] From (11), a_2 is known once d_2 is known, given my assumption on $p(d_2)$. Q^{rf} is also known, according to (2), if $Q(d_2)$ and $p(d_2)$ are known. So $Q(d_2)$ is simulated instead of a_2 and Q^{rf}. π_1 can be obtained if the rest of the variables are known given (7).

Data resource: this research

Based on the MCMC estimates of parameters and latent state variables, I then conduct 10,000 runs of simulations on the policy variables $\{c_1^C, c_1^D, Q(d_2), y_1, \pi_1\}$ across three scenarios. The first is one without debt restructuring, where no perturbation is imposed on the original system. The second scenario under debt relief is one where $\bar{d}_1$ in (8) is reduced exogenously, so that c_1^D there can be increased. Empirically, this is carried out by lowering the mean of $\bar{d}_1$ by 20%, from 0.7462 to 0.5967, in its simulation. The last scenario under extra lending is one where d_2, the debtor country's long-term borrowing, in (8) is raised exogenously, which allows c_1^D to increase potentially. Empirically, the mean of d_2 is dropped also 20%, from 1.2341 to 1.4809, to reflect the exogenous increase of debt burden. The results of simulation under the three scenarios are supplied in Table 3.

The simulated mean of the creditor group's consumption, c_1^C, from the no-restructuring baseline appears to be slightly lower than that from the past, so does the counterpart for the debtor group, c_1^D. However, under the 20% debt relief scenario, creditor group's consumption can be expected to rise by 3.2% from that under the baseline scenario, with the debtor group's consumption falling 1.1% from baseline, or 2.9% from current average. If I perturb the baseline by expanding d_2 by 20%, debtor's consumption would go up by 5.1% from the baseline scenario, or 3.2% from current average, at the expense of creditor's consumption which drops instead by 5.4%, or 8.2% from current average. The debt relief perturbation raises consumption of creditors, but the lending alternative sacrifices the richer economy, which could bring about negative long-term implications.

The baseline simulation produces a value of 0.8858 for $Q(d_2)$, the current value long-term bond issued by the debtor group, which, according to (2), could imply a projected risk-free bond yield of 5.77% given a going recover probability of 0.94. By allowing repeated sampling of system parameters, the MCMC method forecasts a surge of 89 basis points in risk-free bond yields. Alternatively, I could project a drop in recover probability to 0.9312, or 88 basis points, by fixing the risk-free bond yield at that of the creditor group average in Table 1. Under the debt relief scenario, simulation yields a mean of 0.8994 for the risk-free yield, a potential average drop of 94 basis points from the baseline given an accompanying higher recover probability of 0.945. Alternatively under the lending scenario, $Q(d_2)$ is to have a simulated mean of 0.8709 and an implied average risk-free yield of 6.86%, which implies a rise of 109 basis points from the baseline assuming a lower recover probability of 0.935.

Compared against realized output reported in Table 1, the baseline scenario reflects a fall of 9.2% for creditors to only about 81% of frontier, while debtors' output falls by 10.3% to roughly 67% of frontier. Output figures are expected to improve under debt relief to 0.8691 and 0.7439 respectively for creditors and debtors, or 7.3% and 10.6% according improvements from the baseline scenario. Under the lending arrangement, the output is expected to go down, respectively for creditors and debtors, by 1.3% and 2.6% to 0.7995 and 0.6550. They are, however, lower than currently observed figures for creditors and debtors by 9.3% and 12.7% respectively. Adopting sampled parameters by MCMC causes the expectation on output to be considerably lower than currently observed figures, and the prospect is worse in proportion for debtors across all scenarios. Debt relief is the alternative capable of possibly restoring output to the current track. Under the lending arrangement, debtors are expected to suffer an output loss equivalent to a ten-year economic growth.

Inflationary expectation follows a different pattern from the rest of the variables in Table 3 as excessive debt load generates contractionary pressure on output. Baseline inflation for creditors is expected to be 8.9% lower than the observed average within the data period, which is equivalent to an annual rate of 1.65%. For debtors, the expectation falls by 6.2% to an equivalent of 1.85%. Debt relief helps to ease the union-wide economy by just about to restore the currently observed levels, but lending would push creditors' expected inflation down further by 32%, or an annual equivalent of 1.25%. For debtors, inflation is expected to rise by 10.2%, to an annual equivalent of 2.18%. The lending alternative worsens the contractionary effect, especially on creditor countries, leading them to a deflationary prospect. Debt relief, on the other hand, is neutral in this respect.

Table 4 gives country-specific medians of four financial variables, $\{p(d_2), Q^{rf}, T_1^c, \pi_1\}$ under debt relief and extra lending arrangements, based on the 5,000 MCMC runs reported in Table 2 and 3. $p(d_2)$ is calculated each time d_2 is produced in one of the 5,000 runs, and then multiplied by each country's debt to GDP ratio divided by EMU average within the observation period. Country-specific recovery probabilities given d_2 are then obtained by a linear distribution of $exp(0.25)$, based on which the sample mean of $p(d_2)$ is produced. Q^{rf} for a debtor country is according to (2) by the product of $Q(d_2)$ from Table 3 and $p(d_2)$ from above. T_1^c, a creditor country's government taxes, can be obtained by multiplying the group T_1^c by each country's ratio of government consumption to GDP, between 2001 and 2014, normalized by group average within the period. Given y^*, Q^{rf} and T_1^c, π_1 can be obtained for creditor countries according to (7).

The country-specific recovery probabilities for debtors are on average almost three hundred basis points lower than those for creditors. Under a debt relief bail-out, these probabilities are uniformly higher than those under an extra lending bail-out. The average difference for debtors is 247 basis points, and for creditors it is 257 basis points. The magnitude of the difference is, in general, proportional to the relative debt load of each country. According to the analysis, Greece would suffer the biggest drop at 330 basis points. The smallest drop happens to Ireland, one of the debtors, followed by Luxembourg, both slightly below 190 basis points. Nevertheless, the recovery probability of the former is still 531 basis points below that of the latter.

Table 4 Financial Prospects for EMU Countries Given Debt Restructuring Alternatives

Sample Means	$p(d_2)$		Q^{rf}		T_1^C		π_1	
	Relief	*Lending*	*Relief*	*Lending*	*Relief*	*Lending*	*Relief*	*Lending*
					Debtor Countries			
Greece	0.9258	0.8927	0.9077	0.8769				
Italy	0.9325	0.9042	0.9094	0.9030				
Portugal	0.9394	0.9160	0.9115	0.8963				
Belgium	0.9430	0.9223	0.9128	0.9135				
Ireland	0.9458	0.9272	0.9139	0.9040				
Debtor Combined	0.9373	0.9125	0.9110	0.8987				
					Creditor Countries			
Luxembourg	0.9992	0.9803			0.1692	0.1941	1.0050	1.0047
Finland	0.9771	0.9528			0.2265	0.2492	1.0039	1.0016
Netherlands	0.9688	0.9430			0.2626	0.2723	1.0046	1.0033
Germany	0.9632	0.9364			0.1568	0.1998	1.0037	1.0013
Spain	0.9613	0.9342			0.1527	0.1967	1.0054	1.0068
Austria	0.9509	0.9223			0.1691	0.2091	1.0047	1.0038
France	0.9485	0.9195			0.2400	0.2580	1.0038	1.0014
Creditor Combined	0.9670	0.9412			0.1967	0.2256	1.0045	1.0033

Note: This table reports medians for three financial variables, $\{p(d_2), Q^{rf}, T_1^c\}$ under debt relief and extra lending arrangements, based on the 5,000 MCMC runs reported in Table 2 and 3. $p(d_2)$ is calculated each time d_2 is produced in one of the 5,000 runs, and then multiplied by each country's debt to GDP ratio divided by EMU average within the observation period. Country-specific recover probabilities given d_2 are then obtained by a linear distribution of $exp(0.25)$, based on which the sample mean of $p(d_2)$ is produced. Q^{rf} for a debtor country is generated according to (2) by the product of $Q(d_2)$ from Table 3 and $p(d_2)$ from above. T_1^c, a creditor country's government taxes, can be obtained by multiplying the group T_1^c by each country's ratio of government consumption to GDP, between 2001 and 2014, normalized by group average within the period. Given y^*, Q^{rf} and T_1^c, π_1 can be obtained for creditor countries according to (7).

Data source: this research

The comparison on Q^{rf} is reported in Table 4 only for debtor countries. Given a debt relief bail-out, the average implied yields on risk-free bonds are 422 basis points higher than the observed twelve-year average, and 249 basis points above the period between 2011 and 2014. Simulations based on the posterior sampling results have projected a much more risky profile than what markets have actually priced. If a bail-out is carried out alternatively with extra lending, my simulation predicts an even greater loss in the value of outstanding debt, 1.4% in value and 123 basis points in implied yields to be exact. Greece would obviously be the country suffering the most from a switch of debt relief to extra lending at a 307 basis points yield hike. Belgium could actually benefit from the switch, owing much to the fact that its observed yields since 2001 are 46 basis points lower than the twelve-country average.

The country-specific projections on potential fiscal burden are analyzed only for the creditor countries in Table 4, as they would be the ones who need to finance potentially additional government liabilities following any bail-out. The average of T_1^c, as percentages of GDP, is at 19.67%, 825 basis points above the posterior mean given in Table 2. The possible cause is that average factors, which introduce further volatility, are used in achieving the individual projections. However, my focus is in the comparison between the two bail-out alternatives. Lending is a less preferable option as the average fiscal burden could go up by another 289 basis points. Netherlands would be the country impacted the least with an increase of 97 basis points, while Spain would suffer the most as lending would add another 439 basis points of fiscal burden on top of the debt relief option. Actually, the four creditor countries which have the lowest government consumption proportion in GDP, Luxembourg, Spain, Germany and Austria, would be the ones seeing the most increases in their fiscal burden, with around 25% more for the latter three.

The inflationary projection is discussed on an individual country level for creditors to address the deflationary prospects in these countries, following the observation of lower than normal figures under the lending scenario in Table 3. According to Table 4, implied annual inflation is expected to drop 48 basis points annually, from 1.8% to 1.32%, in a migration from debt relief to extra lending. Among all the creditor countries, Germany and France have the highest deflationary risks, with projected annualized inflation rates of 0.53% and 0.56% respectively. In the midst of the deflationary pressure, Spain is expected to maintain an annual inflation of 2.73%, thanks to its long-term observed average of

2.42%. In fact, it is the only creditor country whose inflation is projected to go up, by 58 basis points annually, instead of coming down.

5. A robustness analysis

In the original setup of the model, the zero lower bound of nominal interest is where the currency union would have a recession with output below potential frontier. Recent literature argues, however, once an economy faces a binding zero lower bound on the nominal interest rate, government spending as a stabilization tool is a particularly effective tool as output multiplier of government spending can be much larger than in normal time. After European Central Bank launched its expanded asset purchase program in March 2015, implied yields of government securities in certain countries drop actually below zero, hence formally breaking the bound within EMU.

Table 5 Simulations of EMU Economic Prospects without the Zero Lower Bound

	c_1^c		c_1^D		$Q(d_2)$		y_1		π_1	
	Mean	*Std. err.*	*Mean*	*Std. err.*	*Mean*	*Std. err.*	*Mean*	*Std. err.*	*Mean*	*Std. err.*
					Without Debt Restructuring					
Creditor	0.4971	0.0611					0.8125	0.1087	1.0042	0.0261
Debtor			0.5517	0.0691	0.8977	0.2580	0.7005	0.0894	1.0047	0.0272
					Under 20% Debt Relief					
Creditor	0.4958	0.0609					0.8883	0.1215	1.0046	0.0263
Debtor			0.5493	0.0655	0.9064	0.2613	0.7740	0.0938	1.0051	0.0273
					Under 20% Extra Lending					
Creditor	0.4242	0.0520					0.7901	0.0989	1.0029	0.0250
Debtor			0.5956	0.0742	0.8653	0.2424	0.6044	0.0811	1.0053	0.0274

Note: This table explores the effects of debt restructuring after removing the zero lower bound constraint. If $\bar{d}_1 > \bar{d}_1^*$, then $Q^{rf} \geq 1$ which replaces the previous restriction. $Q(d_2)$ can be derived again from Q^{rf} and $p(d_2)$ according to (2). 10,000 simulations runs on policy variables are then repeated as in Table 3.

Data source: this research

To explore the potential effects of debt restructuring after removing the zero lower bound constraint, I let $Q^{rf} \geq 1$ when $\bar{d}_1 > \bar{d}_1^*$, which replaces the previous restriction on the lower bound of nominal interest rate. Table 5 presents the results of this change, which suggest removing the bound actually improves outcomes of both restructuring alternatives. The simulated mean of the creditor group's consumption, c_1^C, from the no-restructuring baseline would rise by 4.67% to 0.4971, a 1.6% increase from the current average. Its debtor group counterpart, c_1^D falls slightly to 0.5517, a 2% down from current average reported in Table 1. This improves the current situation of creditor sacrificing consumption to subsidize debtor. Under the debt relief scenario, creditor group's consumption would increase by 4.7% from the corresponding figure with zero lower bound in Table 3, while the debtor group's consumption would fall by 1.5% similarly, or a 3.3% drop from current average. Under the extra lending scenario, debtor's consumption would go up by 1.53% from Table 3, or 5.4% from current average, at more expense of creditor's consumption which drops further by 5.6% from Table 3, or 13.3% from current average. In the absence of the lower bound, lending is not restricted, so debt relief is not as effective as before. However, as lending is no longer constrained, extra lending alternative makes things worse by having creditors subsidizing debtors even more.

For $Q(d_2)$, the baseline simulation gives a value 1.3% higher than with the lower bound in Table 3, implying a projected risk-free bond yield of 4.45%, or a drop of 32 basis points. Alternatively, by fixing the risk-free bond yield at that of the creditor group average in Table 1, I could project a drop in recover probability to 0.9438, or an improvement of 38 basis points. Under the debt relief scenario, simulation yields a potential average drop of 79 basis points from Table 3 given an accompanying higher recover probability of 0.945. Alternatively under the lending scenario, $Q(d_2)$ is to have a simulated mean of 0.8653 and an implied average risk-free yield of 7.46%, which implies another increase of 60 basis points from Table 3 assuming a lower recover probability of 0.935.

Compared against realized output reported in Table 1, the baseline scenario reflects a smaller decrease from current average than with lower bound in Table 3, with creditor's output still lagging 8.1% behind, and debtors 6.4%. They are expected to improve under debt relief to be 2.2% higher than in Table 3 for creditors, or 0.78% better than the current average. For debtors, the improvement is 4% from Table 3 or 3.2% from current average. The lending arrangement, however, would lower creditor's output by 1.1% from Table 3, or potentially 11.4% off the current average. The gap for debtors is 7.8% and 19.4% respectively. Without the constraint of interest rate lower bound, excessive lending worsens the debt overhang problem and further depresses output.

The effect of removing interest rate lower bound on inflationary expectation follows the pattern in Table 3 roughly, with slight increases for both groups. Debt relief scenario further raises inflationary expectation, to an annualized 1.85% for creditors and 2.05% for debtors. The lending scenario, however, pushes creditors' expected inflation down further to 1.16% for creditors while raising debtors' expectation up to 2.13%. With the absence of lower bound, creditors' deflationary prospect deteriorates further while debt relief continues to be neutral.

6. Conclusion

This study follows the model of Gali & Monacelli (2008), Roch & Uhlig (2014) and Dogra (2014) to study sovereign debt issues in a currency union. I show in this study, with a Bayesian estimation method, how different the economic and financial prospects could be, within the union, under debt restructuring alternatives. In the case of European Monetary Union, excessive sovereign debt above the Maastricht Convergence Criteria has been in certain countries for years. Debt, economic and financial problems feed consequences to one another in a 'perverse feedback loop', in IMF's term. As parameters in this system are not only time-varying but also involved in sequential relations with other variables, a state-space framework as that in Roch & Uhlig (2014) is necessary. So I choose to estimate the model with an MCMC method, which samples both the parameters and latent states.

I examine the optimality of sovereign debt restructuring alternatives by comparing economic and financial consequences of debt relief versus extra lending. Results of the study suggest upfront debt relief works better than extra lending as the latter runs the risk of breaking up the union. I show in the analysis that, given the sovereign debt problem in Eurozone, the Bayesian approach projects an even more worrisome prospect. Nevertheless, debt relief is one choice for bail-out that is superior to extra lending. Consumption, output and market value of debt are all lower under the latter option. More importantly, compared to debt relief, the lending alternative divides the union further. Debt recovery probability, debt yields, fiscal burden and inflationary expectation separate debtor countries more apart from the creditor ones. Even within the creditor group, extra lending would impose the biggest budget hikes on the most frugal states, in addition to subjecting the larger economies to higher deflationary risks.

The results of this study can be a helpful reference for the consideration of future EMU debt restructuring process, or any other similar sovereign debt situations. My results exemplify that variables in this kind of system should be analyzed with particular recognition of their sequential nature, and accompanied consequences. Treating the interactions within the system in a concurrent manner may produce significantly inaccurate estimation results, and thus suboptimal policies.

The setup of the theoretical model still needs further modification to address refinements considered in existing literature. Alternative estimation algorithm, such as replacing FFBS with straight GG or GS, may have to be adjusted accordingly. Further robustness analysis is required on the composition of creditor and debtor groups, changes in the magnitudes of debt relief or extra lending, and additional restructuring alternatives. Potential results based on a maximum likelihood estimation may also have to be collected for comparisons.

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Appendix

Forward Filter Backward Sampling (FFBS)

To apply time series observations in estimating the system defined by (5)~(12), one need to assume an initial condition $p(X^{(0)}|\Theta)$ and a state dynamics $p(X^{(t+1)}|X^{(t)};\Theta)$, with the two combined yielding a measurement on $p(Y^{(t+1)}|X^{(t+1)},X^{(t)};\Theta)$. For FFBS to work, the state vector *X* should either be linear and normal or discrete. I choose the latter to as the linearity assumption is too restrictive for the model considered in the study. After discretizing *X*, I can apply FFBS to make draws of the entire $\{X^{(g)}\}_{g=1}^{T}$ given Θ. For simplicity, Θ is suppressed in notation while explaining FFBS.

Let $O_t=\{Y^{(g)}\}_{g=1}^{t}$ and $p(X^{(t)},X^{(t-1)}|O_t)$ can be obtained recursively in the following forward-filtering steps. First, based on the definition of conditional probability,

$$\begin{aligned}p\left(X^{(t-1)},X^{(t)},X^{(t+1)}\middle|O_t\right)&=p\left(X^{(t-1)},X^{(t)}\middle|O_t\right)p\left(X^{(t+1)}\middle|X^{(t-1)},X^{(t)},O_t\right)\\&=p\left(X^{(t-1)},X^{(t)}\middle|O_t\right)p(X^{(t+1)}|X^{(t)}\end{aligned}\qquad\text{(A.1)}$$

So the joint density of $\{X^{(t-1)},X^{(t)},X^{(t+1)}\}$ can be separated into one containing $\{X^{(t-1)},X^{(t)}\}$ and the other with $\{X^{(t)},X^{(t+1)}\}$ only. The computation of the former, which can be used later in the backward-sampling phase, is trivial as *X* is discrete, while latter is the assumed state dynamics. (A.1) characterizes how the observation of O_t together with the joint density of $\{X^{(t-1)},X^{(t)},X^{(t+1)}\}$ can help computing $p\left(X^{(t-1)},X^{(t)}\middle|O_t\right)$ recursively. Next, I can move one step forward to realize, by the definition of conditional probability, that

$$\begin{aligned}p\left(X^{(t)},X^{(t+1)}\middle|O_{t+1}\right)&\propto p\left(X^{(t+1)},X^{(t)},Y^{(t+1)}\middle|O_t\right)\\&=p\left(X^{(t)},X^{(t+1)}\middle|O_t\right)p(Y^{(t+1)}|X^{(t)},X^{(t+1)},O_t)\\&=p\left(X^{(t)},X^{(t+1)}\middle|O_t\right)p(Y^{(t+1)}|X^{(t)},X^{(t+1)}).\end{aligned}\qquad\text{(A.2)}$$

Both $p\left(X^{(t)},X^{(t+1)}\middle|O_t\right)$ and $p(Y^{(t+1)}|X^{(t)},X^{(t+1)})$ are known, so $p\left(X^{(t)},X^{(t+1)}\middle|O_{t+1}\right)$ can be computed. The recursion in the previous step $p\left(X^{(t-1)},X^{(t)}\middle|O_t\right)$ is pushed forward one step further to $p\left(X^{(t)},X^{(t+1)}\middle|O_{t+1}\right)$, which can be repeated to cover the entire chain.

The backward-sampling phase attempts to utilize the result above to draw the entire state *X* from

$$p(X^{(t)}|X^{(t+1)},\dots,X^{(T)},O_t)= p\left(X^{(T)},X^{(T-1)}\middle|O_t\right)\prod_{t=T-2}^{1}p\left(X^{(t)}\middle|X^{(t+1)},\dots,X^{(T)},O_T\right), \quad \text{(A.3)}$$

where

$$p(X^{(t)}|X^{(t+1)},\dots,X^{(T)},O_t)= p(X^{(t)}|X^{(t+1)},Y^{(T+1)},O_t) \quad \text{(A.4)}$$

due to the Markov chain property that $X^{(t)}$ is independent of $\{X^{(t+2)},\dots,X^{(T)},Y^{(t+2)},\dots,Y^{(T)}\}$, condition on $\{Y^{(T+1)},X^{(t+1)}\}$. As $\{Y^{(T+1)},O_t\}$ is equivalent to O_{t+1}, the right hand side of (A.4) is known according to (A.2). Based on (A.3), given observation at time t and all subsequent state values $\{X^{(t+1)},\dots,X^{(T)}\}$ obtained from the forward-filtering phase, $X^{(t)}$ can be then drawn in a backward fashion for all *t*.

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